Financing Efficiency of Securities-Based Crowdfunding*

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Abstract

Securities-based crowdfunding is characterized by (i) all-or-nothing financing, (ii) scarce investor profits, and (iii) non-cooperative investors. Privately informed investors internalize all-or-nothing financing as a hedge: good projects are likely to be funded and bad projects are likely canceled, i.e., a loser's blessing. Investors internalize scarce profits as a winner's curse: of funded projects, good projects are likely to be split among many investors and bad projects are likely to be split among fewer. Both the loser's blessing and the winner's curse adversely affect financing efficiency. If investors are not pivotal, efficiency breaks down completely and financing outcomes reflect no information.

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1 Introduction

An entrepreneur requires a minimum level of seed capital when undertaking a new venture. Consequently, seed capital campaigns are inherently "all-or-nothing"; if the entrepreneur is successful in acquiring the minimum level of capital needed, the venture is undertaken, otherwise the venture is canceled. When attempting to secure seed capital, the entrepreneur often taps on the shoulders of many, at times diverse, investors, e.g., family, friends, and angel investors. As such, investors form an informal syndicate and the syndicate's ability to coordinate the members' collective information and actions may be limited due to fragmentation. Moreover, without the ability to coordinate, members of the syndicate may act non-cooperatively to compete for scarce expected profits.

In this paper, we study a model of early venture financing in which (i) financing is all-or-nothing, (ii) financing requires multiple, non-cooperative investors, and (iii) investor profits are scarce. We interpret any financing setting characterized by these three features as securities-based crowdfunding.[1] Our analysis focuses on the financing efficiency achieved via crowdfunding. Including multiple investors creates a rich information environment and a larger crowd collectively possesses better information about a venture's unobservable quality, i.e., the "wisdom of the crowd." However, we show that non-cooperative actions and coordination frictions erode the benefit of greater information. In fact, the erosion in financing efficiency is so severe that a large crowd exhibits no "wisdom" and instead acts collectively uninformed.

The sources for financing inefficiency are threefold. The first and second sources of inefficiency are both driven by the fact that investor participation is (weakly) correlated with the project's unobservable quality, i.e., all else equal, more investors participate in good projects and fewer participate in bad projects. As such, the first inefficiency is due to investors' greater exposure to good projects via the all-or-nothing threshold — if the project is bad, it is less likely that enough

[1] While "crowdfunding" is typically associated with capital campaigns conducted via the internet, it has been a method of capital fundraising for centuries. For example, donation-based crowdfunding has existed since at least the 19th century when the base for the Statue of Liberty was financed by 160,000 donors after the government failed to provide funding. http://www.bbc.com/news/magazine-21932675

investors participate and thus more likely that the project is canceled. This asymmetric exposure relates to what we coin the *loser's blessing*: because investors are somewhat hedged against bad projects, investors respond with more aggressive investment strategies in equilibrium. At times, the loser's blessing leads investors to contribute to projects that they believe are actually bad. The second inefficiency is due to investors' greater exposure to bad projects via limited investment profits — if the project is good, many investors participate and their claims to the project are small and if the project is bad, only a few investors participate and their claims are large. This asymmetric exposure relates to the winner's curse and, in equilibrium, investors respond with more conservative investment strategies, at times abstaining from investing in projects that they believe are actually good. Both the loser's blessing and the winner's curse are associated with a social cost as they distort investors' expected payoffs and investors' investment strategies. Finally, the third source of financing inefficiency is investors' inability to credibly share their private information. Without the ability to share private information, a coordination cost results and financing efficiency is dampened as some valuable projects are forgone and some poor projects are undertaken.

A direct application of our model is to internet (securities-based) crowdfunding in which all-or-nothing thresholds are mandatory and investor participation decisions are decentralized.[2,3] While securities-based crowdfunding via the internet is in its infancy, it is preceded by reward-based crowdfunding (e.g., KickStarter and Indigogo) and donation-based crowdfunding (e.g., GoFundMe) which attract investors who enjoy private benefits from contributing (Boudreau, Jeppesen, Reichstein and Rullani 2015). Beyond securing necessary financing, both reward-based and donation-based crowdfunding have benefited entrepreneurs by enabling feedback from consumer-investors that aids in product and business development.[4] For example, product pre-sales allow consumers to express

[2]The JOBS Act and the SEC's adoption of Regulation Crowdfunding, which became effective May 16, 2016, have set the stage for the growth of securities-based crowdfunding in the United States. See Stemler (2013) for details regarding the JOBS Act, specifically Title III (the CROWDFUND Act) and see Massolution (2015) for an overview of worldwide crowdfunding activity.

[3]In the United States, all-or-nothing financing mechanisms are mandated by Regulation Crowdfunding for securities-based crowdfunding and have been used in Regulation D filings since 1982. The all-or-nothing financing mechanism is also known as a "provision point mechanism" (Bagnoli and Lipman 1989).

[4]Grüner and Siemroth (2016) and Ellman and Hurkens (2016) model crowdfunding as a way to get consumer

their demands for potential products. Only the popular, and likely profitable, products receive sufficient financing and are produced. In this way, reward-based and donation-based crowdfunding harnesses the wisdom of the crowd to finance only the best projects.[5] While it is tempting to hope that securities-based crowdfunding will also harness the wisdom of the crowd, securities-based campaigns differ from reward-based and donation-based campaigns in an important way: the projects in securities-based campaigns are common value goods while the projects in reward-based and donation-based campaigns are private value goods. Our analysis shows that this difference has first-order consequences on financing outcomes. Securities-based crowdfunding destroys the wisdom of the crowd.

Turning to the details of the model, we consider two crowdfunding settings: pivotal crowdfunding and non-pivotal crowdfunding. In pivotal crowdfunding, a small number of non-cooperative, privately-informed investors choose whether or not to provide capital to an entrepreneur. Each investor considers both the impact of her own investment decision and the expected actions of other investors. In the non-pivotal setting, a large number of non-cooperative, privately-informed investors choose whether or not to participate. Unlike the pivotal setting, investors do not internalize their individual actions and instead choose their investment decisions based on the expected actions of all other investors and the project's characteristics.

In the pivotal crowdfunding setting, we first consider a crowd consisting of two investors. Each investor receives a private signal with precision $\alpha > \frac{1}{2}$ about the project's quality and decides whether or not to participate. If the project is good, the project pays a gross return $\Delta > 1$, otherwise the project is bad and pays a gross return of zero. The project requires c units of capital and each investor can contribute at most $M/2$ units. If both investors participate, they split the project's return. If $M/2 \geq c$, a single investor may finance the project and we term

feedback. Agrawal, Catalini and Goldfarb (2014) and Xu (2017) consider other potential benefits of crowdfunding, including increased trade between investors and entrepreneurs, innovation spill-overs, and better geographic dispersion of capital.

[5]There is also strong evidence from prediction markets and earnings forecast platforms that the information content from many dispersed investors is relatively accurate, i.e., the so-called "wisdom of the crowd." See Wolfers and Zitzewitz (2004) and Da and Huang (2016).

this the *solo* setting. In the solo setting, the winner's curse is in play and investors' equilibrium strategies reflect the possibility of financing a bad project alone or splitting the return of a good project. The asymmetric exposure to project outcomes leads investors to abstain from some projects that they believe are positive valued because the expected payoffs of investing are negative. If $M/2 < c \leq M$, joint participation by investors is required to finance the project. We term this the *joint* setting. In the joint setting, the winner's curse does not exist as any funded project requires joint participation. The loser's blessing, however, is present. Investors rationally anticipate that they both must participate to finance the project, and in equilibrium their mutual best responses lead them to invest with nonzero probabilities in projects that they believe are negative valued.

To analyze the efficiency of securities-based crowdfunding, we outline two welfare benchmarks. The first-best benchmark corresponds to the surplus generated by a single monopolist if he possessed all investor capital and observed investors' private signals. The second-best benchmark corresponds to the surplus generated by investors if they could commit to participation strategies that maximize joint surplus (as opposed to individual payoffs). We define coordination costs as the differences between the first-best and second-best benchmarks, and social costs as the differences between the second-best benchmarks and the non-cooperative surplus. In both the solo and joint settings, financing inefficiencies can be so severe that a lone investor with only a single signal performs better than two investors with two signals.

We extend the pivotal crowdfunding setting to $N > 2$ investors to show that financing inefficiency increases with N. Moreover, as N increases, the difference between the first-best and second-best benchmarks shrinks and the coordination cost tends to zero. As such, for large N, financing inefficiencies are due almost entirely to the social costs attributed to the loser's blessing and the winner's curse.

In the non-pivotal setting, there is a unit continuum of investors and a single investor's decision to participate has no impact on whether or not the project is financed. By the strong law of large numbers, the coordination cost is equal to zero — the crowd collectively possesses a perfect signal of a project's quality. Despite the potential for first-best financing efficiency, the social cost is so severe that investment outcomes reflect no information. In equilibrium, either all projects are financed or no projects are financed — regardless of type.

As an example, suppose each investor in the continuum may contribute M units of capital. If each investor followed her signal and signals were accurate with 75% probability, good projects would raise $0.75M$ units of capital and bad projects would raise $0.25M$ units of capital. If the all-or-nothing threshold were between these two values, there would be a perfect loser's blessing and financing would be first-best; good projects would always be financed and bad projects would never be financed. However, this cannot be an equilibrium. An investor observing a bad signal would deviate from following her signal because she expects to earn a risk-less profit. If her bad signal is incorrect she obtains ownership in a good project. Conversely, if her signal is correct, her capital is returned because the project is canceled. Due to the discontinuity in payoffs attributed to the loser's blessing, all investors face the same incentive to deviate and a loser's blessing cannot exist in equilibrium. Therefore, the only equilibria are those in which both good and bad projects are either always financed or never financed. In particular, a range of ex ante positive valued projects are never financed, implying that uninformed investors may outperform an informed crowd.

Our analysis challenges the notion that, like reward-based and donation-based crowdfunding, securities-based crowdfunding will harness the wisdom of the crowd. We show that securities-based crowdfunding will not only fail to aggregate information, but may actually be counterproductive.

1.1 Related Literature

Our paper's focus on fundraising outcomes' partially determining project returns (i.e., the possibility that a project is canceled due to not reaching the all-or-nothing threshold) is related to a growing literature on the real effects of financial markets.[6] In our setting, the combination of

[6]See Bond, Edmans and Goldstein (2012) for a review of the literature. See Dow and Gorton (1997), Dow and Rahi (2003), Dye and Sridhar (2002) and Subrahmanyam and Titman (1999) for theoretical models of financial market feedback. There is also strong empirical support that real decision-makers glean information from secondary trading. Luo (2005) shows that, after announcing an acquisition, managers may cancel the deal if the market responds

private information and the possibility of projects' being canceled leads to information breakdowns. Consequently, our analysis of both the winner's curse and loser's blessing is directly related to the idea that price feedback can hamper the incorporation of information into prices.[7] Goldstein and Guembel (2008), Goldstein, Ozdenoren and Yuan (2013), Edmans, Goldstein and Wei (2015) and Bond, Goldstein and Prescott (2010) provide models in which price feedback either alters investors' trading strategies or obfuscates the signal in prices.[8] In each of these models, active trading markets allow investors to somewhat coordinate and at least partially communicate their information to real decision-makers.

The mechanism for our results differ due to two key features of crowdfunding. First, there is no market in which investors can trade and incorporate their information into prices. In crowdfunding, prices are fixed by entrepreneurs and fundraising quantities convey information.[9,10] Second, all-or-nothing financing creates an asymmetric payoff structure; investors can only profit when they invest.[11] The discrete payoffs that arise due to all-or-nothing financing result in the financing inefficiencies we document.

Goldstein, Ozdenoren and Yuan (2011) and Bond and Goldstein (2015) both show that infor-

negatively. Similarly, Jegadeesh, Weinstein and Welch (1993) and Michaely and Shaw (1994) provide evidence from IPOs that outsiders have incremental information about firm value relative to insiders. Finally, Chen, Goldstein and Jiang (2007) show that firms with more informationally-efficient stock prices demonstrate stronger investment sensitivity to stock price.

[7]While not directly relying on prices, the analysis of informational cascades in Bikhchandani, Hirshleifer and Welch (1992) features individuals who may ignore their own information. In their setting, this is due to learning in a sequential-action game, while in our simultaneous-action setting, asymmetric payoffs lead investors' actions to diverge from their beliefs regarding project quality.

[8]See also Axelson and Makarov (2016) for related work on adverse outcomes when real decisions are made based on fundraising outcomes. In Axelson and Makarov (2016)'s model, an auction's informational efficiency can be destroyed when the information is subsequently used to make an investment decision.

[9]Entrepreneurs can set a fixed price for an offering, or can create a price schedule based on capital raised. In either case, price is established prior to the offering and cannot be adapted in response to information revealed in the fundraising process.

[10]A fundraising process without an all-or-nothing feature is similar to a price as it aggregates dispersed information (Hellwig 1980, Grossman 1976, Hayek 1945). If the fundraising process did not have an all-or-nothing-feature, investors would allocate capital if their information implied the project was worthwhile and would otherwise abstain.

[11]The asymmetric payoff structure separates our analysis from the voting literature, in which collective actions take place when there are enough votes in favor of a proposal. While investors can be pivotal in both settings, only those who contribute to a crowdfunding campaign are committed to the action (investment). For analyses of information aggregation in voting settings, see e.g., Feddersen and Pesendorfer (1997), Feddersen and Pesendorfer (1998) and Bond and Eraslan (2010).

mational inefficiencies can be mitigated when decision makers are willing and able to commit to actions that may run against their best interest. In our model, the social costs associated with the winner's curse and loser's blessing arise because investors cannot commit to socially-optimal decision rules. Our analysis of the second-best benchmark provided by a social planner illustrates the ill effects of individual investors' incentives and inabilities to coordinate or pre-commit.

Our analysis of crowdfunding is also related to IPOs, a setting in which underwriters aggregate information from investors during the bookbuilding process.[12] Similar to Rock (1986), we show that the winner's curse leads to fundraising distortions. However, in our setting, not all projects are financed and the possibility of project cancellation leads to a loser's blessing. Our results of information breakdowns are somewhat surprising in light of the IPO literature which details how to aggregate information from informed investors via bookbuilding. While bookbuilding models of Benveniste and Spindt (1989) and Benveniste and Wilhelm (1990), among others, show that allocation and pricing schedules can be used to elicit investors' information, those papers depend on two important assumptions that are violated in our setting. First, allocation discretion is not common in crowdfunding. Second, small average contributions result in extremely dispersed ownership, likely preventing entrepreneurs from learning individuals' signals, except through aggregate fundraising activity.[13] Bookbuilding models rely on allocation discretion and communication with investors to reward informative bids and effectively aggregate information.[14] Furthermore, it is often assumed in IPO models that adequate financing is available. Failing to acquire adequate financing at any price is common.[15,16] Taken together, these key differences between crowdfunding

[12]See Ritter and Welch (2002), Ljungqvist (2007) and Ritter (2011) for reviews of the literature.

[13]Most investor contributions on crowdfunding platforms are small. Mollick (2014) documents an average investment size of $64 and proposed SEC rules set maximums on individual investments. See https://www.congress.gov/bill/112th-congress/senate-bill/2190/text.

[14]Sherman and Titman (2002) considers the number of investors invited to bid on an IPO, and like our analysis, shows that underwriters may want to limit the size of the invited syndicate.

[15]While less than 20% of firms that begin the IPO filing process ultimately withdraw (Busaba, Benveniste and Guo 2001), about two-thirds of crowdfunding campaigns fail to reach their target (Vulkan, Åstebro and Sirra 2016).

[16]Edelen and Kadlec (2005) consider IPO withdrawals by modeling firms that balance the surplus generated from aggressive pricing against the probability of failed offerings. As in other bookbuilding models, underwriters have allocation discretion, dampening the loser's blessing present in our model.

and IPOs prevent traditional bookbuilding methods from being used to elicit information.

Finally, our work also relates to a nascent theoretical literature on crowdfunding. Hakenes and Schlegel (2014) models endogenous information production in the presence of the winner's curse and all-or-nothing financing thresholds, showing that too much information production may result. Our analysis shows the opposite occurs when crowds become large — breakdowns in financing efficiency would remove all incentives to produce information. Li (2017) considers the optimal financial contract in crowdfunding, showing that typical pro rata allocations are suboptimal in aggregating investors' private information. We are agnostic towards the optimal security design and instead we examine the positive implications of investment efficiency given standard features of crowdfunding. Cumming, Leboeuf and Schwienbacher (2015) compares keep-it-all versus all-or-nothing financing, showing that keep-it-all mechanisms are better for small, scalable projects. We only consider all-or-nothing financing, given its prominence in practice and requirement for US securities-based crowdfunding.

2 Base Model

There exist two risk neutral investors and a project controlled by a benevolent entrepreneur who does not suffer from any agency conflicts. Each investor possesses scarce investment capital.

The project is financially valued according to its quality $F \in \{G, B\}$, its upfront cost c (which is equivalent to its scale), and its promised gross rate of return $\Delta > 1$ (net return $\delta > 0$):

$$V = \mathbb{1}_G \Delta c - c, \tag{1}$$

where $\mathbb{1}_G$ equals one when $F = G$. The project's quality is not observable, but with equal probability, will be good, $F = G$, or bad, $F = B$, i.e.,

$$\Pr(F = G) = \Pr(F = B) = \frac{1}{2}. \tag{2}$$

The project's upfront cost and fundraising goal is $c \in \mathbb{R}^{++}$. To finance the project, initial owners sell 100% of the project's cash flows via equity claims.[17]

Assumption 1. *If the project does not raise an amount weakly greater than c, the project is canceled and committed capital (if any) is returned to investors, i.e., financing is* ***all-or-nothing****.*

The all-or-nothing threshold of c could be exogenously imposed, or the entrepreneur controlling the startup may credibly commit to canceling the project if after "testing the waters" he does not attract at least c dollars of investor interest.

Each of the two investors can contribute a maximum of $\frac{M}{2} > 0$ dollars to the project, so that the total pool of available capital equals M. Prior to making an investment decision, each investor receives a conditionally i.i.d. private signal regarding the project's type and the signal is correct with probability α. Specifically, investors receive a binary signal $\hat{F} \in \{\hat{G}, \hat{B}\}$ and,

$$\Pr(G|\hat{G}) = \Pr(B|\hat{B}) = \alpha > \frac{1}{2}. \tag{3}$$

Assumption 2. *If the project raises c from investors (i.e., it is "funded"), returns are allocated on a* ***pro rata basis****.*[18]

According to assumption 2, if both investors contribute $\frac{M}{2}$ dollars of capital, they each receive a 50% claim to the project's return. If only one investor contributes capital and funds the project, she receives 100% claim to the project's return. We assume that,

$$M \geq c, \tag{4}$$

so that the analysis is nontrivial (i.e., it is possible for the two investors to fund the project).

[17] With two possible states $\{G, B\}$ and the value of the project equaling zero in bad states, it is equivalent to consider the financial claims as being debt.

[18] It is equivalent to assume that the project generates a net return of δ on c dollars and zero net return on any excess quantity greater than c.

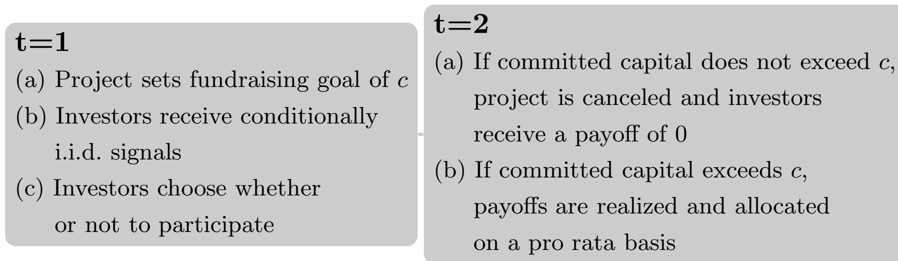


Figure 1: Investment game model timing.

Assumption 3. *Investors cannot credibly share their signals and make their investment decisions simultaneously.*

Assumption 3 represents that entrepreneurs often assemble patchwork networks of investors for early venture financing.

In what follows, we consider the Bayesian Nash Equilibrium to the simultaneous-move investment game. When possible, we focus on non-zero participation equilibria. We only highlight zero participation equilibria when no non-zero participation equilibria exist. While the two investors employ identical strategies in equilibrium, we denote the strategy of investor i as, $\vec{\pi}_i = \{g_i, b_i\}$, where $g_i \in [0,1]$ and $b_i \in [0,1]$ are investor i's mixed strategies based on signals of $\hat{G}$ and $\hat{B}$ respectively.[19] The model's timing is summarized in Figure 1.

The level of the financing threshold can lead to two settings. If $M/2 \geq c$, a single investor may finance the project and we term this the *solo* setting (denoted with the superscript S). If $M/2 < c \leq M$, joint participation by investors is required to finance the project. We term this the *joint* setting (denoted with the superscript J).

First, consider the solo setting, i.e., $\frac{M}{2} \geq c$. Investor i's expected payoff (as a return on invested

[19] While we explicitly restrict investors to contributing the full amount of their capital, relaxing this assumption does not change the main insights of the model. Whenever investors play non-deterministic strategies, there are infinite potential equilibria that satisfy investors' binding participation constraints with less than 100% contributions, and each equilibria gives rise to the same financing probabilities in each state, thus giving identical financing efficiency.

capital), conditional on observing $\hat{G}$, is equal to,

$$\begin{aligned}\Pi(\hat{G}|\vec{\pi}_i^S, \vec{\pi}_{-i}^S) =& \alpha^2\delta\left(g_i^S g_{-i}^S \frac{1}{2} + g_i^S(1-g_{-i}^S)\right)\\ &+\alpha(1-\alpha)\delta\left(g_i^S b_{-i}^S \frac{1}{2} + g_i^S(1-b_{-i}^S)\right)\\ &-(1-\alpha)\alpha\left(g_i^S b_{-i}^S \frac{1}{2} + g_i^S(1-b_{-i}^S)\right)\\ &-(1-\alpha)^2\left(g_i^S g_{-i}^S \frac{1}{2} + g_i^S(1-g_{-i}^S)\right). \end{aligned} \tag{5}$$

The expression in (5) is intuitive: when investor i observes $\hat{G}$, she believes that the project is good with probability α. Furthermore, conditional on the project's being good, investor i believes that investor $-i$ will also observe a signal of $\hat{G}$ with probability α. Thus, conditional on investor i observing $\hat{G}$, her state probability that (i) the project is good and (ii) investor $-i$ observes $\hat{G}$ as well is equal to α^2. In this state, she chooses a mixed strategy of g_i^S and rationally expects investor $-i$ to choose a mixed strategy of g_{-i}^S. Therefore, she expects to split the project's return with probability $g_i^S g_{-i}^S$, expects to be the lone investor with probability $g_i^S(1-g_{-i}^S)$, and does not participate (earning zero) with probability $(1-g_i^S)$. The first line in (5) represents these possible outcomes,

$$\underbrace{\alpha^2}_{\substack{\text{Project is type } G\\ \text{Both investors observe } \hat{G}}} \delta\left(\underbrace{g_i^S g_{-i}^S \frac{1}{2}}_{\text{Both investors invest}} + \underbrace{g_i^S(1-g_{-i}^S)}_{\text{Only investor } i \text{ invests}}\right).$$

A similar analysis applies to (i) the state in which investor i is correct while investor $-i$ observes the incorrect signal $\hat{B}$; (ii) the state in which she is incorrect while investor $-i$ observes the correct signal $\hat{B}$; and (iii) the state in which she is incorrect and investor $-i$ also observes the incorrect signal $\hat{G}$.

Investor i's expected payoff, conditional on observing a signal of $\hat{B}$, is equal to,

$$\begin{aligned}\Pi(\hat{B}|\vec{\pi}_i^S, \vec{\pi}_{-i}^S) =&(1-\alpha)^2\delta\left(b_i^S b_{-i}^S \frac{1}{2} + b_i^S(1-b_{-i}^S)\right)\\ &+(1-\alpha)\alpha\delta\left(b_i^S g_{-i}^S \frac{1}{2} + b_i^S(1-g_{-i}^S)\right)\\ &-\alpha(1-\alpha)\left(b_i^S g_{-i}^S \frac{1}{2} + b_i^S(1-g_{-i}^S)\right)\\ &-\alpha^2\left(b_i^S b_{-i}^S \frac{1}{2} + b_i^S(1-b_{-i}^S)\right).\end{aligned} \tag{6}$$

While the expected payoff in (6) is similar to that in (5), it differs in the strategy played by investor i (b_i^S rather than g_i^S), and she believes that the project is good with probability $(1-\alpha)$.

In this non-cooperative game, investors do not coordinate their investment decisions to exploit the informational content of the two signals they possess. Instead, each investor makes her investment decision based on her individual signal and as a best response to the other investor's strategy. The following lemma provides the symmetric strategies used by the two investors in the solo setting (the subscripts i and $-i$ are suppressed).

Lemma 1. *In a solo setting, the equilibrium investment strategies are,*

$$\begin{cases} g^S = b^S = 1 & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ g^S = 1\ ,\ b^S = 0 & \delta \in \left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right), \frac{\alpha}{1-\alpha}\right) \\ g^S = \frac{2(\alpha(1+\delta)-1)}{2(\alpha(1+\delta)-1)+(1-\alpha^2(1-\delta)-2\alpha\delta)}\ ,\ b^S = 0 & \delta \in \left[\frac{1-\alpha}{\alpha}, \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right)\right) \\ g^S = b^S = 0 & \delta \in \left[0, \frac{1-\alpha}{\alpha}\right). \end{cases} \tag{7}$$

In the solo setting, both investors are willing to invest if δ is sufficiently large, i.e., $\delta \geq \frac{\alpha}{1-\alpha}$. Investors that observe $\hat{B}$ are willing to invest because the return is large enough to overcome their belief that the project will payoff with probability $(1-\alpha)$. For smaller values of δ, the relevant thresholds become slightly more nuanced. No investor that observes $\hat{B}$ is willing to invest if δ is smaller than $\frac{\alpha}{1-\alpha}$ because she expects the project to be negative valued. An investor that observes $\hat{G}$ has greater confidence that the project will yield a positive payoff and will invest with probability one if, $\delta \geq \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right)$. At first blush, it may seem surprising that the threshold is not equal to $\frac{1-\alpha}{\alpha}$; the threshold at which an investor that observes $\hat{G}$ believes that the project itself is weakly positive valued. Instead, the threshold is larger by a factor of,

$$\frac{1+\alpha}{2-\alpha} > 1 \qquad \forall \quad \alpha \in \left(\frac{1}{2}, 1\right]. \tag{8}$$

A larger return is required due to the *winner's curse.* Specifically, in a solo setting, investors are asymmetrically exposed to project outcomes due to pro rata allocations. If an investor observes $\hat{G}$ she expects that the project is (i) good with probability α and (ii) that it is likely that the other investor observed $\hat{G}$. Thus, she will likely split the project's return if the project is indeed revealed to be good. If, however, her signal is incorrect, she is more likely to fund an entire bad project by herself. Thus, she is less exposed to good projects when she invests and more exposed to bad projects. This implies that she requires a return greater than $\frac{1-\alpha}{\alpha}$ to invest with probability one. Therefore, the winner's curse may lead to investment distortions in which investors observing $\hat{G}$ act almost as if they observed a bad signal.

For promised returns smaller than $\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right)$, an investor that observes $\hat{G}$ will not invest with probability one. Below this threshold, the probability of investing is declining in δ and equals zero at,

$$\delta = \frac{1-\alpha}{\alpha},$$

which is the return that makes the *project's* expected value equal to zero for an investor that observes $\hat{G}$.

Now, consider the joint setting in which $\frac{M}{2} < c$ and $M \geq c$. Due to the all-or-nothing feature, investor i's expected payoff, conditional on observing $\hat{G}$ is equal to,

$$\begin{aligned}\Pi(\hat{G}|\vec{\pi}_i^J, \vec{\pi}_{-i}^J) =& \alpha^2 \left(g_i^J g_{-i}^J \frac{\delta}{2} \right) + \alpha(1-\alpha) \left(g_i^J b_{-i}^J \frac{\delta}{2} \right) \\ & -(1-\alpha)\alpha \left(g_i^J b_{-i}^J \frac{1}{2} \right) - (1-\alpha)^2 \left(g_i^J g_{-i}^J \frac{1}{2} \right). \end{aligned} \tag{9}$$

The expression in (9) is similar to that outlined in (5). However, the payoffs in the states in which one investor participates and the other does not are set equal to zero. In the states in which both investors do not invest, the project does not attract sufficient capital and is canceled, yielding a payoff equal to zero for both investors (committed capital, if any, is returned). Requiring joint investor participation implies that the winner's curse is not a concern. Investor i's expected payoff conditional on observing $\hat{B}$ is similar to that in (6), except that the states in which only one investor participates are set to zero.

$$\begin{aligned}\Pi(\hat{B}|\vec{\pi}_i^J, \vec{\pi}_{-i}^J) =& (1-\alpha)^2 \left(b_i^J b_{-i}^J \frac{\delta}{2} \right) + (1-\alpha)\alpha \left(b_i^J g_{-i}^J \frac{\delta}{2} \right) \\ & -\alpha(1-\alpha) \left(b_i^J g_{-i}^J \frac{1}{2} \right) - \alpha^2 \left(b_i^J b_{-i}^J \frac{1}{2} \right). \end{aligned} \tag{10}$$

The following lemma provides the symmetric strategies used by the two investors in the joint setting (the subscripts i and $-i$ are suppressed).

Lemma 2. *In a joint setting, the equilibrium investment strategies are,*

$$\begin{cases} g^J = b^J = 1 & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ g^J = 1 \ , \ b^J = \frac{(1-\alpha)\alpha(\delta-1)}{(1-\alpha)\alpha(\delta-1)+\alpha(1+\delta)-\delta} & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ g^J = 1 \ , \ b^J = 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right) \\ g^J = b^J = 0 & \delta \in \left[0, \frac{(1-\alpha)^2}{\alpha^2}\right). \end{cases} \tag{11}$$

According to Lemma 2, both investors will participate in the joint setting if δ is sufficiently large. Furthermore, the threshold δ that compels both investors to participate corresponds to the same threshold in the solo setting, i.e., $\frac{\alpha}{1-\alpha}$. Below the threshold return of $\frac{\alpha}{1-\alpha}$, an investor that observes a signal of $\hat{B}$ will employ a nondeterministic strategy as long as $\delta \geq 1$. Intuitively this relates to the *loser's blessing*. Because both investors are required to fund the project due to all-or-nothing financing and, ceteris paribus, good projects merit more investor participation, investors are asymmetrically exposed to projects' returns. Unlike the winner's curse which implies the asymmetry weights bad projects more heavily, the loser's blessing weights good projects more heavily. All else equal, this makes investors more willing to invest in projects. In fact, for project returns in the region $\left[1, \frac{\alpha}{1-\alpha}\right)$, there is positive probability that an investor observing $\hat{B}$ will invest, despite that investor believing the project itself is negative valued. The threshold at which no investor that observes $\hat{B}$ is willing to invest is $\delta = 1$. Intuitively, this is the threshold that a single investor with no information would use. Therefore, the loser's blessing may lead to investment distortions in which investors observing $\hat{B}$ act almost as if they observed a good signal.

In the following section, we explore the welfare costs associated with inefficient crowdfunding. Specifically, we decompose the welfare losses into two components: (i) a coordination cost due to investors' inability to coordinate their private information and (ii) a social cost due to investors' acting to maximize their individual payoffs rather than social welfare.

2.1 Welfare Analysis

Both the winner's curse and loser's blessing lead to investment efficiency distortions. To examine these investment distortions, we define a welfare metric termed *value-add*,

$$v = \Pr(\text{Funded})\left(\Pr(G|\text{Funded})\delta - \Pr(B|\text{Funded})\right). \tag{12}$$

The value-add metric considers the implications of both investing and not investing. The term within parenthesis in (12) corresponds to the welfare effects of investing: conditional on investment, welfare goes up when the project is good and goes down when it is bad. The term within parenthesis is then scaled by the probability of investing. The value-add metric can be thought of similarly to that of a portfolio return on the set of all available projects — the expected return on any selected project is $\Pr(G|\text{Funded})\delta - \Pr(B|\text{Funded})$ and the probability that a given project is financed is $\Pr(\text{Funded})$.

The first-best (denoted FB) value-add corresponds to our welfare metric if a single monopolist had M units of capital and received two conditionally i.i.d. signals regarding the project's type. The first-best value-add serves as an upper bound: the non-cooperative investors cannot outperform the monopolist. We adopt the notation $N, \underline{n}$ as a subscript on value-add denoting the total number of investors N and the number of investors required for investment $\underline{n}$. For example, the solo setting value-add is denoted $v_{2,1}$, and the joint setting value-add is denoted $v_{2,2}$. We focus only on the projects with $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right]$, as the investment criteria and value-add metrics are trivial outside of this range.

Proposition 1. *In a two investor setting, the solo setting value-add is equal to the first-best value-add for* $\delta \in \left[1, \frac{\alpha}{1-\alpha}\right]$, *otherwise the solo setting value-add is strictly less than the first-best value-add,*

$$v_N^{FB} - v_{2,1} = \begin{cases} > 0 & \delta \in \left(\frac{\alpha}{1-\alpha}, \frac{\alpha^2}{(1-\alpha)^2}\right) \\ = 0 & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right] \\ > 0 & \delta \in \left(\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{13}$$

The joint setting value-add is equal to the first-best value-add for $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right]$, *otherwise the joint setting value-add is strictly less than the first-best value-add,*

$$v_N^{FB} - v_{2,2} = \begin{cases} > 0 & \delta \in \left(1, \frac{\alpha^2}{(1-\alpha)^2}\right) \\ = 0 & \delta \in \left(\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{14}$$

Solo and joint investment efficiency underperforms relative to first-best according to the value-

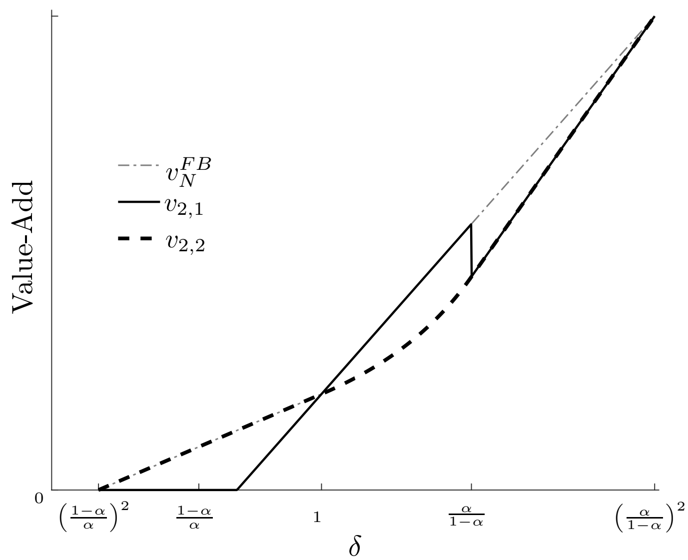


Figure 2: Value-add metrics relative to first-best: the horizontal axis represents a project's promised return and the vertical axis measures value-add. The first-best, solo and joint value-add metrics are depicted. For projects with $\delta < 1$, the solo value-add (the solid line) appears below both the joint value-add (the dashed line) and the informed monopolist benchmark. The opposite holds for $\delta > 1$. The discontinuity at $\delta = \frac{\alpha}{1-\alpha}$ for the solo value-add is due to investors observing $\hat{B}$ switching between investing with probability zero below $\frac{\alpha}{1-\alpha}$ and investing with probability one above it.

add metric. Figure 2 shows that the solo setting underperforms when $\delta < 1$ and $\delta > \frac{\alpha}{1-\alpha}$, while the joint setting underperforms when $\delta > 1$. The welfare inefficiencies implied by Proposition 1 and Figure 2 are due to two sources. First, welfare losses may come from investors' inability to coordinate their private signals — *a coordination cost*. Second, welfare losses may also come from investors acting in their own interest rather than what is socially optimal — *a social cost*.

To decompose welfare losses into coordination costs and social costs, we outline a second-best value-add metric. The second-best (denoted SB) value-add corresponds to the maximum joint surplus that can be achieved when investors cannot coordinate signals but can commit to participation strategies. The second-best benchmark is equivalent to a setting in which a social planner dictates a strategy $\{g^{SB}, b^{SB}\}$ and that strategy maximizes joint surplus.

Comparing v_N^{FB} to $v_{N,\underline{n}}^{SB}$ provides a measure of the coordination cost,

$$\text{Coordination Cost} = v_N^{FB} - v_{N,\underline{n}}^{SB}. \tag{15}$$

The coordination cost is the difference in value-add from a monopolist that observes all private signals and the value-add of investors whose participation strategies maximize joint surplus (as opposed to individual payoffs). Similarly, comparing $v_{N,\underline{n}}^{SB}$ to $v_{N,\underline{n}}$ provides a measure of the social cost,

$$\text{Social Cost} = v_{N,\underline{n}}^{SB} - v_{N,\underline{n}}. \tag{16}$$

The social cost is the difference in value-add of investors whose strategies maximize joint surplus and the value-add of investors who maximize their individual payoffs.

Corollary 1.1. *In a two investor setting, the social cost in the solo setting is strictly positive for* $\delta \in \left[\frac{1-\alpha}{\alpha}, 1\right)$ *and* $\delta \in \left[\frac{\alpha}{1-\alpha}, \frac{\alpha^2}{(1-\alpha)^2}\right]$*, otherwise the social cost equals zero,*

$$v_{2,1}^{SB} - v_{2,1} = \begin{cases} > 0 & \delta \in \left(\frac{\alpha}{1-\alpha}, \frac{\alpha^2}{(1-\alpha)^2}\right) \\ = 0 & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right] \\ > 0 & \delta \in \left(\frac{1-\alpha}{\alpha}, 1\right) \\ = 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right]. \end{cases} \tag{17}$$

The social cost in the joint setting is equal to zero for all $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right]$*,*

$$v_{2,2}^{SB} - v_{2,2} = \begin{cases} = 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right]. \end{cases} \tag{18}$$

According to Corollary 1.1, the welfare losses in solo financing are due partly to a social cost. In fact, a comparison of Proposition 1 to Corollary 1.1 shows that for projects with $\delta \in \left(\frac{1-\alpha}{\alpha}, 1\right)$ and $\delta \in \left(\frac{\alpha}{1-\alpha}, \frac{\alpha^2}{(1-\alpha)^2}\right)$, solo financing is associated with both a coordination cost and a social cost. The coordination cost is straightforward; because a single investor can finance the project, the inability to coordinate information increases the chances that a bad project is financed. The social cost is due to investor's individual payoff functions being different than the aggregate payoff function. Specifically, investors are asymmetrically exposed to project payoffs due to the winner's curse.

For joint financing, non-cooperative investors perform equal to second-best. The welfare losses with joint financing are entirely due to a coordination cost; because both investors are required to finance the project, the inability to coordinate information increases the chances that a good project is not financed. Corollary 1.1 suggests that there is not a social cost associated with the loser's blessing. The result is because an investor's individual payoff function is proportional to the aggregate payoff function (by a factor of 1/2). However, we show in Section 2.2 that there is a social cost associated with the loser's blessing for larger crowds. As long as all investors are not required to finance the project, the loser's blessing leads investors to employ strategies that are different from what a social planner would prescribe.

2.1.1 Two Investors versus One Investor

Given the inefficiencies induced by the two non-cooperative investors, it is natural to compare solo and joint financing to a setting in which there is only one investor with one signal. That is, how does one investor with a single signal and M units of capital compare to two non-cooperative investors each possessing a private signal?

Corollary 1.2. *For projects with $\delta < 1$, non-cooperative solo investing performs worse than a single investor. For projects with $\delta > 1$, non-cooperative joint investing performs worse than a single investor.*

On one hand, moving from a single investor to two increases the total information available. On the other hand, moving from a single investor to two produces a welfare loss due to the coordination cost and social cost. According to Corollary 1.2, for projects with $\delta < 1$ (ex ante negative valued) that can be financed by a sole investor, the welfare loss exceeds the benefit of more information.

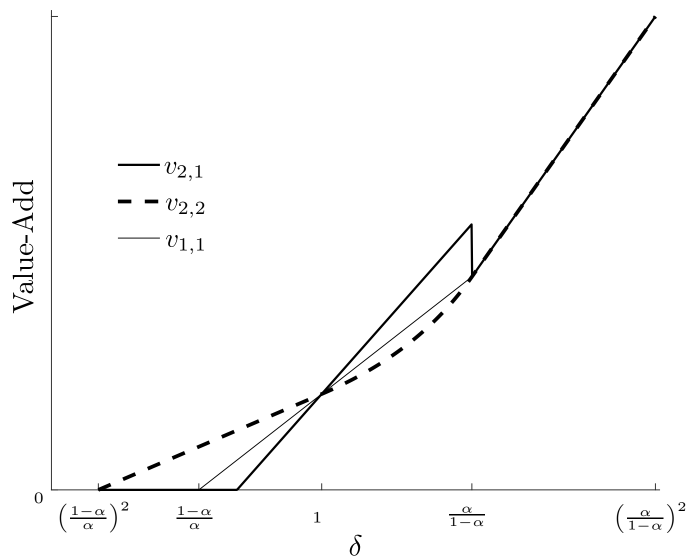


Figure 3: Value-add metrics relative to an single investor setting: For projects with $\delta < 1$, the single investor setting weakly outperforms solo financing. For projects with $\delta > 1$, the single investor setting weakly outperforms joint financing.

For projects with $\delta > 1$ (ex ante positive valued) that require joint financing, the welfare loss also exceeds the benefit of more information. The results of Corollary 1.2 are depicted in Figure 3, further highlighting the welfare costs associated with non-cooperative syndicates. In the figure, the horizontal axis represents a project's promised return and the vertical axis measure value-add. The solo and joint financing value-add metrics are depicted as the bold lines and the single investor value-add metric is the solid thin line. The solo value-add metric (the solid bold line) falls below the single investor value-add for projects with $\delta < 1$. The joint value-add metric (the dashed bold line) falls below the single investor value-add for projects with $\delta > 1$.

The welfare analysis in this section highlights that inefficiencies may be so strong that financing outcomes would improve if there were only one investor with one signal. Taken together, the findings suggest that the coordination and social costs due to the winner's curse and the loser's blessing are exacerbated as the number of investors increases.

2.2 Large Crowds

In this extension, we consider a setting with a finite $N > 2$ investors. We maintain the base model's assumptions regarding project values, types and the precision of investors' signals. We also maintain Assumptions 1, 2, and 3 — financing is all-or-nothing, project returns are allocated on a pro rata basis, and investor cannot coordinate. To adapt our model to more than two investors, we assume M dollars are available from N investors, so each can invest $\frac{M}{N}$. Let n be the total number of investors who contribute to a project, noting that n also indexes the realized financing state. The probability of realizing state n depends on investors' strategies. We use the superscript LC to signify the pivotal-investor, large-crowd setting. We denote the strategy of investor i as, $\vec{\pi}_i^{LC} = \{g_i^{LC}, b_i^{LC}\}$, where $g_i^{LC} \in [0,1]$ and $b_i^{LC} \in [0,1]$ are investor i's mixed strategies based on signals of $\hat{G}$ and $\hat{B}$ respectively. We denote all other investors' strategies, not including investor i, as $\vec{\pi}_{-i}^{LC}$. Given the project cost c, at least,

$$\underline{n} \equiv \frac{cN}{M}, \tag{19}$$

investors must participate for the project to be financed. In state n, n investors contribute $\frac{c}{n}$ dollars to the project ($n\frac{M}{N} - \frac{c}{n}$ is returned), and will receive payoffs $\frac{V}{n}$ if the project is good.

Investor i will contribute capital as long as her expected payoff from doing so is weakly positive. Following a good signal, investor i will contribute capital if:

$$\Pi_i(\hat{G}|\vec{\pi}_i^{LC}, \vec{\pi}_{-i}^{LC}) = \sum_{n=\underline{n}-1}^{N-1} \left(\alpha \Pr(n|G, \vec{\pi}_{-i}^{LC}, N-1, \alpha)\frac{V-c}{n} - (1-\alpha)\Pr(n|B, \vec{\pi}_{-i}^{LC}, N-1, \alpha)\frac{c}{n} \right) \geq 0, \tag{20}$$

and will contribute capital after receiving a bad signal if:

$$\Pi_i(\hat{B}|\vec{\pi}_i^{LC}, \vec{\pi}_{-i}^{LC}) = \sum_{n=\underline{n}-1}^{N-1} \left((1-\alpha) \Pr(n|G, \vec{\pi}_{-i}^{LC}, N-1, \alpha)\frac{V-c}{n} - \alpha\Pr(n|B, \vec{\pi}_{-i}^{LC}, N-1, \alpha)\frac{c}{n} \right) \geq 0. \tag{21}$$

Several features of (20) and (21) are worth highlighting. First, only those states in which the

Table 1: Example equilibria of discrete agent model with $N = 20$, $\underline{n} = 15$, $\delta = 1$ and $\alpha = 0.75$.

	Non-Cooperative (1)	Second-Best (2)	First-Best (3)
g^{LC}	1	1	
b^{LC}	0.530	0.368	
$E[\Pi\|G]$	0.037	0.040	
$E[\Pi\|B]$	0	0.004	
Value-Add	0.369	0.441	0.491

aggregate financing equals or exceeds c are included in the summations due to the all-or-nothing feature. Second, the summations are indexed from $\underline{n} - 1$ to $N - 1$, as from the perspective of investor i, her investment will provide the additional capital to meet the threshold (thus, she is pivotal). Third, for states with more aggregate financing, investor i's shares are diluted. Finally, investors' equilibrium strategies determine $\Pr(n|G, \vec{\pi}_{-i}^{LC}, N-1, \alpha)$ and $\Pr(n|B, \vec{\pi}_{-i}^{LC}, N-1, \alpha)$.

To solve the model, we first establish two results that simplify the search for equilibrium strategies, and then use numerical methods to find investors' mixing probabilities.[20] The first result is that all investors play identical strategies. If this were not the case, then either (20) or (21) would not bind for some investor i, which cannot be an equilibrium. The second result is that if investors play an internal mixing probability after observing one type of signal, then investors' mixing probability must be a corner solution after observing the other type of signal. The intuition is that internal mixing probabilities imply that either (20) or (21) binds, and thus the other condition is either satisfied strictly, or does not hold. Using these results, we use numerical methods to solve for the probabilities that set (20) or (21) equal to zero, giving investors' equilibrium strategies.

Table 1 provides an example equilibrium to illustrate both the coordination cost and social cost associated with the loser's blessing. The example highlights a high threshold, $\underline{n} = 15$ and $N = 20$. The example is also compared to both the first-best outcome (when a monopolist observes

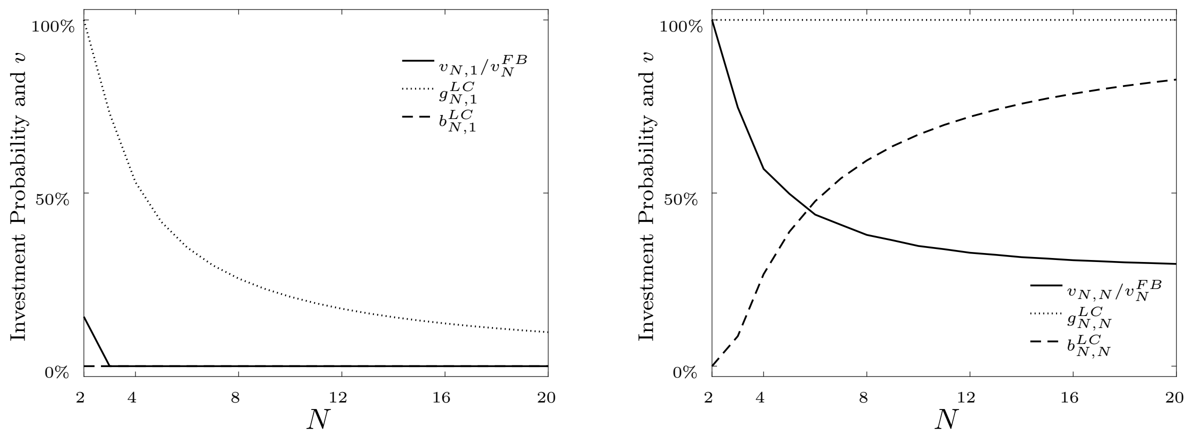
[20]Appendix B provides details on our solution strategy and numerical methods.

all signals) and the second-best outcome (when investors commit to strategies that maximize joint surplus). Before examining the equilibrium strategies, note that coordination costs and individual incentive distortions affect total value-add. Comparing columns (2) and (3), the second-best value-add measures is smaller than the first-best value-add, signifying coordination costs. Comparing columns (1) and (2), the second-best value-add is greater than the non-cooperative value-add. This implies that individual incentives are distorting investment decisions (relative to the social optimum) leading to a social cost.

Turning to the individual strategies, column (1) shows the influence of the loser's blessing. Because bad projects would be unlikely to meet a high threshold if investors simply followed their signals, investors observing bad signals have an incentive to invest; in this case, 53% of the time. For these investors, the expected payoff from investing is zero. Individual investors increase their investment probability as long as it is profitable to invest, and in doing so, collectively compete away any profit opportunities for those observing a bad signal. However, column (2) shows that the second-best strategies result in expected profits to investors that observe both good and bad signals. The second-best strategies prescribe that investors observing bad signals invest 37% of the time. Without their occasional investment, good projects would be funded less often, although financed projects would be more likely good quality. Ultimately, the second-best strategies trade off ex-post financing quality with ex-ante financing probability, balancing the forces to maximize value-add.

The example equilibrium provides some insight into investors' equilibrium strategies and their performance relative to a social planner. In particular, the example highlights that both coordination costs and social costs from inefficient individual investment decisions harm financing efficiency. The following analysis considers how individual investment strategies and financing efficiency depend on the size of the syndicate.

Figure 4 displays equilibrium strategies for investors under two scenarios that correspond to our base model analysis. The left panel shows the equilibrium strategies when only one investor



(a) 1 investor required.

(b) N investors required.

Figure 4: Investors' equilibrium strategies for $\delta = 0.5$ and $\alpha = 0.75$.

is required to meet the threshold, corresponding to the solo financing setting. The horizontal axis represents the number of investors N and the vertical axis represents the probability of investing. The probability of investing based on a bad signal, $b_{N,1}^{LC}$, is equal to zero because an investor observing $\hat{B}$ believes the project is negative valued and knows it will be funded if she participates. The probability of investing based on a good signal, $g_{N,1}^{LC}$, is decreasing with the number of investors N. This effect is due to a worsening winner's curse. As investors are added, the distributions for the number of investors receiving good signals for good and bad projects separate, increasing the severity of the asymmetric exposure to bad projects.

The right panel of Figure 4 shows the equilibrium strategies when all investors are required to meet the threshold, corresponding to the joint financing setting. When all investors are required to finance the project, the loser's blessing drives investors' strategies. As N grows, it becomes less likely that an investor will be pivotal and investors become more willing to ignore their information. Accordingly, we see that the probability of investing conditional on a bad signal increases with N (the probability of investing based on a good signal equals one). While these two panels depict particular sets of parameters, they show the general feature that as the number of investors increases, the winner's curse and loser's blessing are both exacerbated. In other words, individual

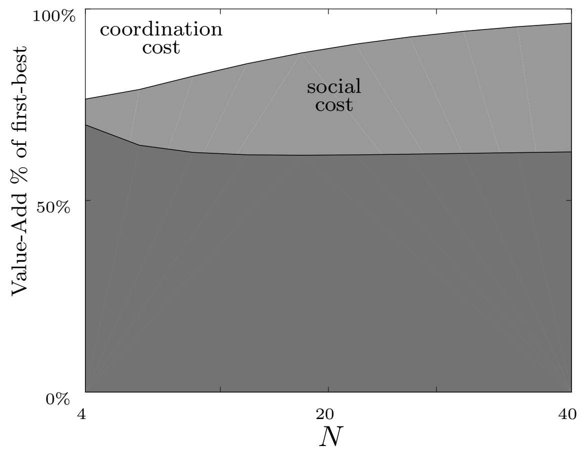


Figure 5: Non-cooperative and second-best value-add metrics (relative to first-best) as functions of the number of investors N. The unshaded region represents the coordination cost due to investors' inability to coordinate their private information. The lighter shaded region represents the social cost due to non-cooperative investors acting to maximize their individual payoffs rather than the aggregate surplus. The darkest shaded region represents the value-add captured by non-cooperative investors. The non-varying parameters are $\underline{n} = \frac{N}{4}$, $\alpha = 0.75$, and $\delta = 1.5$.

investors' incentives lead them to increasingly ignore their information.

In both panels of Figure 4, as N increases and investors make less use of their information, overall financing efficiency decreases. The solid line in both panels represents the non-cooperative value add $v_{N,\underline{n}}$ as a fraction of v_N^{FB}. As N increases, non-cooperative value-add declines. Including a greater number of investors impedes financing efficiency due to greater social costs attributed to the winner's curse when $\underline{n} = 1$ and the loser's blessing when $\underline{n} = N$.

Figure 5 decomposes the welfare losses associated with changes in N when 25% of investors are required to finance a project ($\underline{n} = \frac{N}{4}$). The welfare losses are decomposed into the coordination cost component and the social cost component. The top line represents the second-best value-add as a fraction of first-best. The bottom line represents the non-cooperative value-add as a fraction of first-best. The unshaded area between first-best (i.e., 100%) and second-best value-add depicts the coordination cost. The lighter shaded area between second-best and non-cooperative value-add depicts the social cost.

Figure 5 shows that N can have opposite effects on coordination cost and social cost. The coordination cost is shrinking in N; as N gets large the first-best and second-best value-add benchmarks converge. When participation strategies are socially optimal, larger N allows for better separation between good and bad projects. However, larger N leads to more distortion to individuals' strategies, so social cost is increasing in N. As such, financing inefficiencies are almost entirely due to the social cost for large N. In the next section, we consider a setting in which there is a continuum of investors and no single investor is pivotal. We show that financing inefficiency suffers and financing outcomes reflect no information.

3 Non-pivotal Investors

We extend our venture financing model to a setting of many atomistic investors. We maintain all assumptions regarding the project and its payoffs from Section 2. The entrepreneur solicits contributions from a unit continuum of risk neutral investors. Each investor can invest at most M dollars into the project so that the total available capital equals M. We assume that $c \leq M$ so that financing the project is possible.

As in the base model, investors simultaneously participate on the crowdfunding platform. Investors that observe $\hat{G}$ invest with probability $g^{NP} \in [0,1]$ and investors that observe $\hat{B}$ invest with probability $b^{NP} \in [0,1]$ (NP denotes non-pivotal crowdfunding). While all investors employ identical strategies in equilibrium, we denote the strategy of investor i as, $\vec{\pi}_i^{NP} = \{g_i^{NP}, b_i^{NP}\}$. Furthermore, we denote the strategy of all other investors, not including investor i, as, $\vec{\pi}_{-i}^{NP} = \{g_{-i}^{NP}, g_{-i}^{NP}\}$.

Financing outcomes depend on individual investors' decisions. Investors weigh potential payoffs against explicit project costs as well as dilution expected from other investors. Formally, an investors's participation is determined by the sign on,

$$\Pr(G|\hat{F})\sigma(G)\delta - \Pr(B|\hat{F})\sigma(B), \tag{22}$$

in which the share of investors' committed capital deployed to the project is,

$$\sigma(F) \equiv \frac{c}{M\ell(F)}\mathbb{1}_F. \tag{23}$$

$\ell(F)$ is the measure of participating investors, which depends on α and investors' strategies. $\mathbb{1}_F$ is an indicator function that equals one if $\ell(F)M \geq c$, reflecting that financing is all-or-nothing and projects that receive insufficient capital are canceled. $\sigma(G)$ therefore represents each investor's expected investment in good projects and $\sigma(B)$ represents expected investment in bad projects. In many cases, $\sigma(G) < \sigma(B)$, meaning investors are more exposed to bad projects due to the winner's curse. With a continuum of investors, there also exists the possibility of a loser's blessing. If a loser's blessing exists, it perfectly screens projects. That is, when the loser's blessing is in place, bad projects are not financed and $\sigma(B) = 0$. As will be seen shortly, both the winner's curse and the loser's blessing influence the participation strategies of investors.

Because investor i is atomistic, she is not pivotal and she does not internalize the impact of her investment decision on whether or not the project is funded. Therefore, her participation decision is dictated by the sign on the expression in (22). Taking all other investors' equilibrium participation strategies as given, investor i will invest with probability one if the expression in (22) is strictly positive. If the expression equals zero, investor i is indifferent between participating and not participating and she may choose to mix between her options. Investor i abstains from investing if the expression in (22) is strictly negative.

If investor i observes $\hat{G}$, the expression in (22) is explicitly given by,

$$\overbrace{\alpha}^{\Pr(G|\hat{G})}\underbrace{\left(\frac{c\mathbb{1}_G}{M(\alpha g_{-i}^{NP} + (1-\alpha)b_{-i}^{NP})}\right)}_{\sigma(G)}\delta M - \overbrace{(1-\alpha)}^{\Pr(B|\hat{G})}\underbrace{\left(\frac{c\mathbb{1}_B}{M((1-\alpha)g_{-i}^{NP} + \alpha b_{-i}^{NP})}\right)}_{\sigma(B)} M, \tag{24}$$

where $\mathbb{1}_G$ is an indicator function that equals one if $M\left(\alpha g_{-i}^{NP} + (1-\alpha)b_{-i}^{NP}\right) \geq c$ and $\mathbb{1}_B$ is an indicator function that equals one if $\left((1-\alpha)g_{-i}^{NP} + \alpha b_{-i}^{NP}\right) \geq c$, reflecting whether fundraising successfully met the all-or-nothing threshold. Similarly, if investor i observes $\hat{B}$ instead, the expression in (22) is explicitly given by,

$$(1-\alpha)\left(\frac{c\mathbb{1}_G}{M(\alpha g_{-i}^{NP}+(1-\alpha)b_{-i}^{NP})}\right)\delta M-\alpha\left(\frac{c\mathbb{1}_B}{M((1-\alpha)g_{-i}^{NP}+\alpha b_{-i}^{NP})}\right)M. \tag{25}$$

If $\mathbb{1}_B = 0$, then there would exist a perfect loser's blessing; a type B project would never reach its financing goal of c. As such, investor i would be perfectly hedged against bad projects and it would be dominant for her to choose $g_i^{NP} = b_i^{NP} = 1$ as the expressions in (24) and (25) would be strictly positive. Likewise, all other investors would also find a strategy of $g_{-i}^{NP} = b_{-i}^{NP} = 1$ optimal. If all investors always invest, then both types of projects would attract M units of capital (each investor in the unit continuum participates). Therefore, if a bad project cannot be financed it must be the case that a good project also cannot be financed, i.e., $\mathbb{1}_G = 0$. Because $c \leq M$, our analysis implies that a loser's blessing cannot exist in equilibrium.

Lemma 3. *A loser's blessing never exists in equilibrium.*

Lemma 3 says that a loser's blessing cannot occur in equilibrium. For a loser's blessing to exist, there must exist a wedge between the quantities of capital that good and bad projects raise. Additionally, the project's scale c must lie within this wedge so that only a good project achieves its fundraising goal while a bad project fails. A necessary condition for a loser's blessing is that investor participation varies depending on the project's unobservable quality. If, however, a loser's blessing exists in equilibrium, all investors would find participation optimal. Thus, the equilibrium would unravel and investor participation for both types of projects would be full participation, eliminating the financing wedge.

Before proceeding with the remainder of our analysis, it is useful to divide projects according to scale c. If investors commit to following their signals (which may not be an equilibrium strategy), the minimum quantity of capital that can be raised is,

$$\underline{K} \equiv (1-\alpha)M, \tag{26}$$

and the maximum quantity of capital that can be raised is,

$$\overline{K} \equiv \alpha M. \tag{27}$$

The preceding capital boundaries, $\underline{K}$ and $\overline{K}$, are useful in characterizing investors' equilibrium strategies and the resulting financing efficiency. To ease exposition, we define small-scale, medium-scale, and large-scale projects using these boundaries.

Definition 1. *Define* ***small-scale projects*** *as projects with* $c \in (0, \underline{K}]$*,* ***medium-scale projects*** *as projects with* $c \in (\underline{K}, \overline{K}]$*, and* ***large-scale projects*** *as projects with* $c \in (\overline{K}, M]$*.*

In what follows, we provide lemmas which divide the characterization of the simultaneous-move Bayesian Nash Equilibrium strategies according to the small-scale, medium-scale, and large-scale boundaries, highlighting that different tensions arise according to the scale of the project.

Lemma 4. *For small-scale projects, the equilibrium investment strategy for* $\delta \in [1, \infty)$ *is,*

$$\begin{cases} g^{NP} = b^{NP} = 1 & \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ g^{NP} = 1 \ , \ b^{NP} = 0 & \left[1, \frac{\alpha}{1-\alpha}\right) . \end{cases} \tag{28}$$

For $\delta \in [0, 1)$ *any strategy* $\vec{\pi} = \{g^{NP}, b^{NP}\}$ *with* $g^{NP}, b^{NP} \in [0, 1]$ *is an equilibrium strategy if and only if no projects are financed, i.e.,*

$$M(\alpha g^{NP} + (1-\alpha) b^{NP}) < c, \tag{29}$$

$$M((1-\alpha) g^{NP} + \alpha b^{NP}) < c. \tag{30}$$

Small-scale projects with ex ante positive values (i.e., $\delta \geq 1$) will be financed. Enough investors will observe good signals, regardless of true project type, to meet the financing threshold. Ex ante negative valued projects (i.e., $\delta < 1$) will not be financed. While a good signal indicates positive conditional value, the winner's curse exactly offsets the informational advantage provided by the signal, leading to no projects' being financed. Because more investors will receive good signals for a good project, investors will be more heavily rationed in good projects.[21]

Lemma 5. *For medium-scale projects and large-scale projects, the equilibrium investment strategy for $\delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right)$ is,*

$$g^{NP} = b^{NP} = 1 \tag{31}$$

For $\delta \in \left[0, \frac{\alpha}{1-\alpha}\right)$ any strategy $\vec{\pi} = \{g^{NP}, b^{NP}\}$ with $g^{NP}, b^{NP} \in [0,1]$ is an equilibrium strategy if and only if no projects are financed, i.e.,

$$M(\alpha g^{NP} + (1-\alpha)b^{NP}) < c, \tag{32}$$

$$M((1-\alpha)g^{NP} + \alpha b^{NP}) < c. \tag{33}$$

Proposition 2. *In equilibrium, either (i) all projects are financed regardless of quality or (ii) no projects are financed.*

Proposition 2 highlights that, with a continuum of investors, the crowd's information cannot be utilized; either all projects or no projects are financed. Figure 6 characterizes the financing and no-financing regions based on project scale (c) and promised returns (δ). At the top of the figure, all projects with high promised returns, $\delta > \frac{\alpha}{1-\alpha}$, are financed regardless of quality. The return is sufficiently high that investors observing bad signals are willing to invest. At the bottom of the figure, no projects with low promised returns, $\delta < 1$, are financed. Due to the winner's

[21] Note that for projects with $\delta \in \left[1, \frac{\alpha}{1-\alpha}\right)$, financing outcomes perfect reveal project quality ex post. This suggests entrepreneurs may be able to use the realized capital attracted to cancel bad projects. However, if the entrepreneur committed to canceling projects only raising $(1-\alpha)M$, a loser's blessing would exist and all investors would participate, and capital attracted would no longer be a valuable signal.

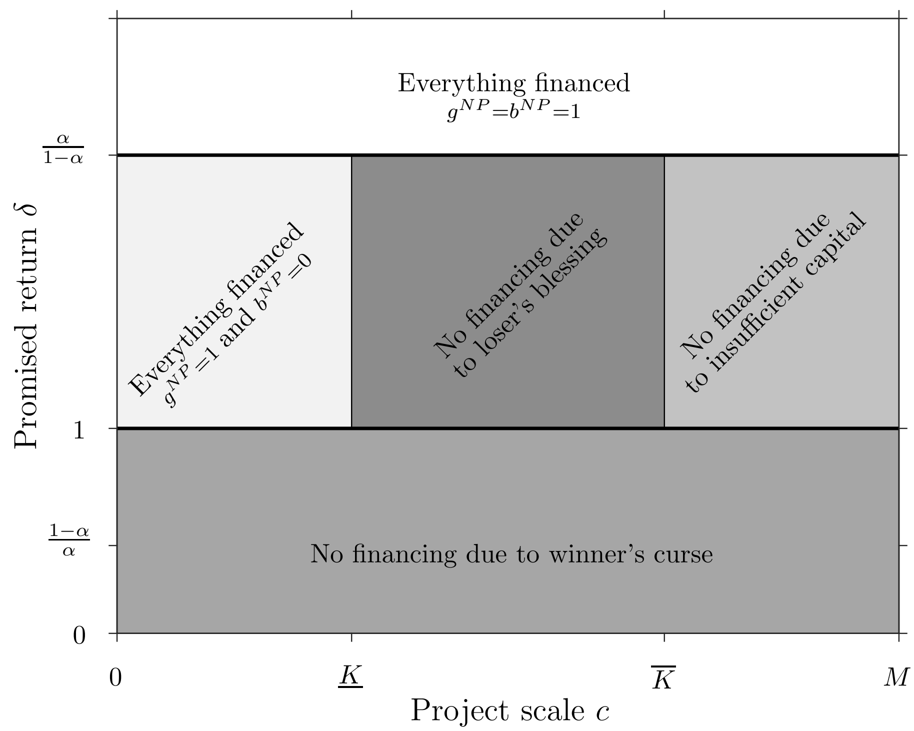
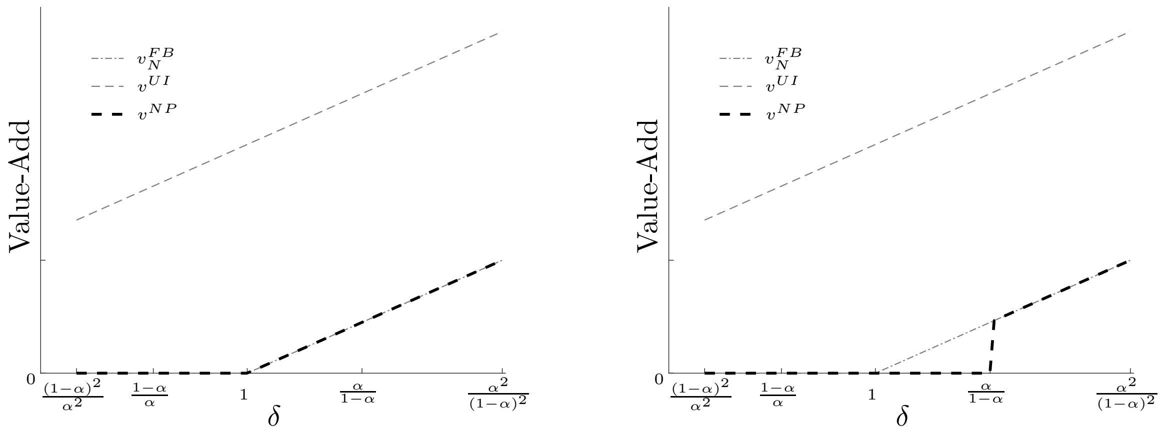


Figure 6: Non-pivotal crowdfunding financing regions. Projects with sufficiently high promised returns are always financed in equilibrium and projects with low promised returns are never financed in equilibrium. Financing outcomes only depend on a project's promised return δ and not on unobservable project quality.

curse, even investors receiving good signals are not willing to invest. For projects with moderate promised returns, either all projects or no projects are financed. Small-scale projects are always financed, as even a bad project has sufficiently many investors observing (incorrect) good signals. However, above $\underline{K}$, a bad project will not attract sufficiently many investors to be financed. As a result, if investors followed their signals, a loser's blessing would exist. However, this cannot be an equilibrium, and the only possible equilibria involve no financing of any projects. As a result, medium and large-scale projects are never financed for moderate levels of promised returns.[22]

[22]The financing disparity between small-scale and medium and large-scale projects suggests that entrepreneurs may endogenously set their all-or-nothing thresholds artificially low in order to influence financing success. Most importantly, increasing the proportion of small-scale projects will not lead to efficient financing, as all small-scale projects are financed regardless of project quality. Furthermore, while moral hazard or asymmetric information may lead entrepreneurs to manipulate all-or-nothing thresholds, the cost associated with setting a higher threshold



(a) Value-add for crowdfunded small-scale projects v^{NP} compared to first-best v_N^{FB} and a setting with no information v^{UI}.

(b) Value-add for crowdfunded medium- and large-scale projects v^{NP} compared to first-best v_N^{FB} and a setting with no information v^{UI}

Figure 7: Value-add metrics in the crowdfunding setting.

From a welfare perspective, the crowdfunding setting underperforms a setting in which investors have no information (in which the relevant participation threshold would be $\delta = 1$). There are two forces at play that cause this inefficiency. One, the rationality of investors eliminates the possibility of a loser's blessing to exist. Therefore, either all projects or no projects are financed in equilibrium. Second, in any equilibria in which all medium- and large-scale projects are financed, the required return is larger than what would be required in a setting in which agents have no information. Specifically, because investors have *some* information, they will not rationally invest after receiving a bad signal unless the return is sufficiently high. Thus, for all investors to participate, they require a higher promised return, $\delta \geq \frac{\alpha}{1-\alpha}$ (small-scale projects perform equally as well when investors have no information, i.e., all projects with $\delta \geq 1$ are financed).

Figure 7 shows the financing inefficiency for small-scale projects (left panel) and medium and large-scale projects (right panel). For small-scale projects, the value-add is equal to what uninformed investors would earn (uninformed investors are denoted by UI). For medium and large-scale projects, the value-add is equal to uninformed value-add for low and high-promised-return projects.

likely varies based on project quality, allowing thresholds to serve as signaling mechanisms. We leave the analysis of signaling in crowdfunding to future research.

However, for moderate-promised-return projects, the crowdfunding setting strictly underperforms uninformed investors. Because ex ante positive valued projects are not financed in crowdfunding, value-add is left on the table and thus uninformed investors would perform better.[23] Since no projects are financed in this region, ex ante and ex post measures of welfare decline in α.

4 Concluding Discussion

Our analysis highlights tensions in financing efficiency when (i) financing is all-or-nothing, (ii) profits are scarce, and (iii) investors are dispersed and non-coordinating. While the winner's curse is well understood by financial economists, we highlight the loser's blessing and its role in eroding financing efficiency. So long as all-or-nothing financing thresholds result in project cancellation and investor participation is correlated with project quality, investors internalize the implicit hedge coming from the loser's blessing and ignore negative information. All-or-nothing financing leads investors to make less use of bad signals (due to the loser's blessing), pro-rata allocations lead investors to make less use of good signals (due to the winner's curse), and financing is inefficient.

While we model a single-period financing game to focus attention on the sources of financing inefficiency, we expect the same economic forces to apply to similar multiple-period games. For example, investors would wait to contribute until the last period as long as financing cannot be secured in earlier periods, essentially reducing the multiple-period game to a one-period game. Conversely, if investors were incentivized to contribute early via first-come, first-served allocations, the same would apply to the first-period of a multiple-period game. In both cases, multiple-period games reduce to one-period games in which the winner's curse and loser's blessing affect investors' participation strategies.[24] Incorporating heterogeneous investors could mitigate these forces in a

[23]While the continuum setting results in zero welfare for moderate-return, medium-scale projects, the same is not true in the N-agent setting, i.e., the limit of the N-agent model is not equal to the model of the limit. With a finite number of investors, each investor internalizes that there is *some* probability that she may be pivotal and she responds with very precise mixed strategies. The strategies aggregate to higher funding probabilities for good projects relative to bad projects, resulting in higher financing efficiency.

[24]On many platforms, contributions can be withdrawn prior to the end of the campaign. In such cases, all investors would optimally contribute in the first period, and would make their ultimate decisions in the final period by either

multiple-period game, and this extension is left to future research.

Our analysis suggests that crowds of strictly-profit-motivated investors are unlikely to provide significant capital to early stage ventures due to inefficient financing. However, rewards-based and donations-based crowdfunding have grown successfully based on contributions from individuals who receive private benefits. To the extent that profit-motivated investors also enjoy private benefits from a project, e.g. access to a new local brewery, a portion of investors may be able to commit to following their signals, restoring some degree of financing efficiency. By attracting consumer-like investors, securities-based crowdfunding platforms may be able to efficiently finance projects by enticing investment with the promise of future returns (having a common value), while relying on investors' preferences (their private values) to reveal their true beliefs.

maintaining or withdrawing their investments.

References

Agrawal, Ajay, Christian Catalini and Avi Goldfarb. 2014. "Some Simple Economics of Crowdfunding." *Innovation Policy and the Economy, University of Chicago Press* 14(1):63–97.

Axelson, Ulf and Igor Makarov. 2016. "Informational Black Holes in Auctions." *Working Paper*.

Bagnoli, Mark and Barton L. Lipman. 1989. "Provision of Public Goods: Fully Implementing the Core through Private Contributions." *Review of Economic Studies* 56(4):583–601.

Benveniste, Lawrence M and Paul A Spindt. 1989. "How Investment Bankers Determine the Offer Price and Allocation of New Issues." *Journal of Financial Economics* 24(2):343–361.

Benveniste, Lawrence M and William J Wilhelm. 1990. "A Comparative Analysis of IPO Proceeds Under Alternative Regulatory Environments." *Journal of Financial Economics* 28(1):173–207.

Bikhchandani, Sushil, David Hirshleifer and Ivo Welch. 1992. "A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades." *Journal of Policial Economy* 100(5):992–1026.

Bond, Philip, Alex Edmans and Itay Goldstein. 2012. "The Real Effects of Financial Markets." *Annual Review of Financial Economics* 4:339–360.

Bond, Philip and Hülya Eraslan. 2010. "Strategic Voting over Strategic Proposals." *Review of Economic Studies* 77(2):459–490.

Bond, Philip and Itay Goldstein. 2015. "Government Intervention and Information Aggregation by Prices." *The Journal of Finance* 70(6):2777–2812.

Bond, Philip, Itay Goldstein and Edward Simpson Prescott. 2010. "Market-Based Corrective Actions." *The Review of Financial Studies* 23(2):781–820.

Boudreau, Kevin J., Lars Bo Jeppesen, Toke Reichstein and Francesco Rullani. 2015. "Crowdfunding as 'Donations': Theory & Evidence." *Harvard Business School Working Paper 16–038* .

Busaba, Walid Y., Lawrence M. Benveniste and Re-Jin Guo. 2001. "The Option to Withdraw IPOs During the Premarket: Empirical Analysis." *Journal of Financial Economics* 60(1):73–102.

Chen, Qi, Itay Goldstein and Wei Jiang. 2007. "Price Informativeness and Investment Sensitivity to Stock Price." *The Review of Financial Studies* 20(3):619–650.

Cumming, Douglas J., Gael Leboeuf and Armin Schwienbacher. 2015. "Crowdfunding Models: Keep-It-All vs. All-Or-Nothing." *Working Paper*.

Da, Zhi and Xing Huang. 2016. "Harnessing the Wisdom of Crowds." *Working Paper*.

Dow, James and Gary Gorton. 1997. "Stock Market Efficiency and Economic Efficiency: Is There a Connection?" *The Journal of Finance* 52(3):1087–1129.

Dow, James and Rohit Rahi. 2003. "Informed Trading, Investment, and Welfare." *The Journal of Business* 76(3):439–454.

Dye, Ronald A and Sri S Sridhar. 2002. "Resource Allocation Effects of Price Reactions to Disclosures." *Contemporary Accounting Research* 19(3):385.

Edelen, Roger M. and Gregory B. Kadlec. 2005. "Issuer Surplus and the Partial Adjustment of IPO Prices to Public Information." *Journal of Financial Economics* 77(2):347–373.

Edmans, Alex, Itay Goldstein and Jiang Wei. 2015. "Feedback Effects, Asymmetric Trading, and the Limits to Arbitrage." *American Economic Review* 105(12):3766–3797.

Ellman, Matthew and Sjaak Hurkens. 2016. "Optimal Crowdfunding Design." *Working Paper*.

Feddersen, Timothy and Wolfgang Pesendorfer. 1997. "Voting Behavior and Information Aggregation in Elections with Private Information." *Econometrica* 65(5):1029–1058.

Feddersen, Timothy and Wolfgang Pesendorfer. 1998. "Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting." *The American Political Science Review* 92(1):23–35.

Goldstein, Itay and Alexander Guembel. 2008. "Manipulation and the Allocational Role of Prices." *The Review of Economic Studies* 75(1):133–164.

Goldstein, Itay, Emre Ozdenoren and Kathy Yuan. 2011. "Learning and Complementarities in Speculative Attacks." *The Review of Economic Studies* 78(1):263–292.

Goldstein, Itay, Emre Ozdenoren and Kathy Yuan. 2013. "Trading Frenzies and Their Impact on Real Investment." *Journal of Financial Economics* 109(2):566–582.

Grossman, Sanford. 1976. "On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information." *The Journal of Finance* 31(2):573–585.

Grüner, Hans Peter and Christoph Siemroth. 2016. "Crowdfunding, Efficiency and Inequality." *Working Paper*.

Hakenes, Hendrik and Friederike Schlegel. 2014. "Exploiting the Financial Wisdom of the Crowd: Crowdfunding as a Tool to Aggregate Vague Information." *Working Paper*.

Hayek, Friedrich August. 1945. "The Use of Knowledge in Society." *American Economic Review* pp. 519–530.

Hellwig, Martin F. 1980. "On The Aggregation of Information in Competitive Markets." *Journal of Economic Theory* 22(3):477–498.

Jegadeesh, Narasimhan, Mark Weinstein and Ivo Welch. 1993. "An Empirical Investigation of IPO Returns and Subsequent Equity Offerings." *Journal of Financial Economics* 34(2):153–175.

Li, Jiasun. 2017. "Profit Sharing: A Contracting Solution to Harness the Wisdom of the Crowd." *Working Paper*.

Ljungqvist, Alexander P. 2007. *IPO Underpricing in: Eckbo, B.E. (Ed.), Handbook of Corporate Finance: Empirical Corporate Finance*. North Holland.

Luo, Yuanzhi. 2005. "Do Insiders Learn From Outsiders? Evidence From Mergers and Acquisitions." *The Journal of Finance* 60(4):1951–1982.

Massolution, CL. 2015. "Crowdfunding Industry Report.".

Michaely, Roni and Wayne H Shaw. 1994. "The Pricing of Initial Public Offerings: Tests of Adverse-Selection and Signaling Theories." *The Review of Financial Studies* 7(2):279–319.

Mollick, Ethan. 2014. "The Dyanmics of Crowdfunding: An Exploratory Study." *Journal of Business Venturing* 29(1):1–16.

Ritter, Jay R. 2011. "Equilibrium in the Initial Public Offerings Market." *Annual Review of Financial Economics* 3:347–374.

Ritter, Jay R. and Ivo Welch. 2002. "A Review of IPO Activity, Pricing, and Allocations." *The Journal of Finance* 57(4):1795–1828.

Rock, Kevin. 1986. "Why New Issues Are Underpriced." *Journal of Financial Economics* 15(1-2):187–212.

Sherman, Ann E. and Sheridan Titman. 2002. "Building the IPO Order Book: Underpricing and Participation Limits with Costly Information." *Journal of Financial Economics* 65(1):3–29.

Stemler, Abbey R. 2013. "The JOBS Act and Crowdfunding: Harnessing the Power – and Money – of the Masses." *Business Horizons* 56:271–275.

Subrahmanyam, Avanidhar and Sheridan Titman. 1999. "The Going-Public Decision and the Development of Financial Markets." *The Journal of Finance* 54(3):1045–1082.

Vulkan, Nir, Thomas Åstebro and Manuel Fernandez Sirra. 2016. "Equity Crowdfunding: A New Phenomena." *Journal of Business Venturing Insights* 5:37–49.

Wolfers, Justin and Eric Zitzewitz. 2004. "Prediction Markets." *Journal of Economic Perspectives* 18(2):107–126.

Xu, Ting. 2017. "Learning from the Crowd: The Feedback Value of Crowdfunding." *Working Paper* .

Appendix A

We establish several useful results before proving Section 2's lemmas and propositions.

Lemma A1. *In both the solo and the joint investment settings, each investor's equilibrium probability of investing is weakly higher conditional on observing $\hat{G}$ relative to the probability conditional on observing $\hat{B}$, i.e., $g_i^S \geq b_i^S$ and $g_i^J \geq b_i^J$.*

Proof of Lemma A1: Suppose not: in a solo setting, there exists an equilibrium in which $g^S < b^S$ for some value δ' (where the notation i and $-i$ are suppressed since strategies are symmetric in equilibrium). First, the expression in (5) simplifies to,

$$\Pi(\hat{G}|\vec{\pi}^S) = g^S \left(\left(\delta\alpha^2 - (1-\alpha)^2\right) \left(1 - \frac{g^S}{2}\right) + \alpha(1-\alpha)(\delta - 1) \left(1 - \frac{b^S}{2}\right) \right), \tag{A1}$$

and the expression in (6) simplifies to,

$$\Pi(\hat{B}|\vec{\pi}^S) = b^S \left(\left(\delta(1-\alpha)^2 - \alpha^2\right) \left(1 - \frac{b^S}{2}\right) + \alpha(1-\alpha)(\delta - 1) \left(1 - \frac{g^S}{2}\right) \right). \tag{A2}$$

Furthermore, at $\delta = \delta'$, $g^S < b^S \leq 1$ implies that $g^S < 1$. Thus, g^S is either an internal value or g^S equals zero. Both imply $\Pi(\hat{G}|\vec{\pi}^S, \delta = \delta') = 0$. $\Pi(\hat{B}|\vec{\pi}^S, \delta = \delta')$, however, is weakly positive,

$$\Pi(\hat{B}|\vec{\pi}^S, \delta = \delta') = b^S \left(\left(\delta'(1-\alpha)^2 - \alpha^2\right) \left(1 - \frac{b^S}{2}\right) + \alpha(1-\alpha)(\delta' - 1) \left(1 - \frac{g^S}{2}\right) \right) \tag{A3}$$

$$\geq 0. \tag{A4}$$

The preceding expression is rearranged to provide a lower bound on δ',

$$\delta' \geq \frac{\alpha(2 - \alpha b^S - (1-\alpha)g^S)}{(1-\alpha)(2 - (1-\alpha)b^S - \alpha g^S)}. \tag{A5}$$

The lower bound is decreasing in b^S,

$$\frac{\mathrm{d}}{\mathrm{d}b^S} \left(\frac{\alpha(2 - \alpha b^S - (1-\alpha)g^S)}{(1-\alpha)(2 - (1-\alpha)b^S - \alpha g^S)} \right) = -\frac{\alpha(2\alpha - 1)(2 - g^S)}{(1-\alpha)(2 - (1-\alpha)b^S - \alpha g^S)^2} \tag{A6}$$

$$\leq 0, \tag{A7}$$

and the bound takes it smallest value at $b^S = 1$,

$$\left.\frac{\alpha(2-\alpha b^S-(1-\alpha)g^S)}{(1-\alpha)(2-(1-\alpha)b^S-\alpha g^S)}\right|_{b^S=1} = 1+\frac{2\alpha-1}{(1-\alpha)(1+\alpha(1-g^S))}. \tag{A8}$$

However, at $\delta = 1+\frac{2\alpha-1}{(1-\alpha)(1+\alpha(1-g^S))}$, the payoff $\Pi(\hat{G}|\vec{\pi}^S)$ is strictly positive for all values of b^S and non-zero values of g^S,

$$\Pi\left(\hat{G}\middle|\vec{\pi}^S, \delta = 1+\frac{2\alpha-1}{(1-\alpha)(1+\alpha(1-g^S))}\right) = \frac{g^S(2\alpha-1)(2-g^S+(1-\alpha)\alpha((1-b^S)+(1-g^S)^2))}{2(1-\alpha)(1+\alpha(1-g^S))}. \tag{A9}$$

Thus, an investor observing $\hat{G}$ strictly increases her payoff by choosing $g^S = 1$ at $\delta = 1+\frac{2\alpha-1}{(1-\alpha)(1+\alpha(1-g^S))}$ and for all larger values of δ. Therefore, there does not exist any value δ' at which $g^S < b^S$.

A similar proof shows that there does not exist any value of δ for which $b^J > g^J$ in joint settings as well. The proof is omitted for the a sake of brevity.

■

Lemma A2. *No equilibrium exists in which both mixed strategies in $\vec{\pi}^S$ ($\vec{\pi}^J$) are internal values. That is, either g^S, b^S or both are deterministic (either g^J, b^J or both are deterministic).*

Proof of Lemma A2: Suppose not: in a solo setting, both g^S and b^S are internal values. Internal values of g^S and b^S imply that, in equilibrium, the following equalities hold,

$$0 = \Pi(\hat{G}|\vec{\pi}^S) = 0 \tag{A10}$$

$$0 = \Pi(\hat{B}|\vec{\pi}^S) = 0. \tag{A11}$$

The preceding system of equations is explicitly given by,

$$0 = g^S\left(\left(\delta\alpha^2-(1-\alpha)^2\right)\left(1-\frac{g^S}{2}\right)+\alpha(1-\alpha)(\delta-1)\left(1-\frac{b^S}{2}\right)\right) \tag{A12}$$

$$0 = b^S\left(\left(\delta(1-\alpha)^2-\alpha^2\right)\left(1-\frac{b^S}{2}\right)+\alpha(1-\alpha)(\delta-1)\left(1-\frac{g^S}{2}\right)\right). \tag{A13}$$

There are four solutions to the system: (i) $\{g^S, b^S\} = \{0, 0\}$, (ii) $\{g^S, b^S\} = \{2, 2\}$, (iii) $\{g^S, b^S\} = \{0, \frac{2(\alpha-(1-\alpha)\delta)}{2(\alpha-(1-\alpha)\delta)-(2\alpha-\alpha^2(1-\delta)-\delta)}\}$, and (iv) $\{g^S, b^S\} = \{\frac{2(\alpha\delta-(1-\alpha))}{2(\alpha\delta-(1-\alpha))+(1-\alpha^2(1-\delta)-2\alpha\delta)}, 0\}$. None of the solutions consist of two internal values.

A similar proof may be conducted in joint settings. The proof is omitted for the sake of brevity.

$\blacksquare$

Proof of Lemma 1: The proof is constructed as follows: first, we solve for the region of δ for which investors are willing to invest regardless of their signals. Second, we solve for the strategy of investors observing $\hat{B}$ for values of δ smaller than those for which investors invest regardless of their signal. Finally, we solve for the strategy of investors observing $\hat{G}$ for values of δ smaller than those for which investors invest regardless of their signal, controlling for the strategy of investors that observe $\hat{B}$.

First, consider the case in which investors always invest regardless of signal, i.e., $\vec{\pi}_i^S = \{1, 1\}$ and $\vec{\pi}_{-i}^S = \{1, 1\}$. For the equilibrium to exist, it must be the case that $\Pi(\hat{B}|\vec{\pi}_i^S, \vec{\pi}_{-i}^S) \geq 0$. The inequality $\Pi(\hat{B}|\vec{\pi}_i^S, \vec{\pi}_{-i}^S) \geq 0$ binds at,

$$0 = \left(\delta(1-\alpha)^2 - \alpha^2\right) + (1-\alpha)\alpha(\delta - 1). \tag{A14}$$

Thus, if,

$$\delta \geq \frac{\alpha}{1-\alpha}, \tag{A15}$$

an equilibrium in which both investors will always invest (regardless of signal) is sustainable. Therefore, $b_i^S = b_{-i}^S = g_i^S = g_{-i}^S = 1$ for $\delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right)$.

Now consider $\delta < \frac{\alpha}{1-\alpha}$. Investor i's probability of investing based on a signal $\hat{B}$ must be less than one. By Lemmas A1 and A2, if b^S is an internal value, it must be that g^S is equal to one. Here, because the outside option is zero, the mutual best responses that implicitly define b_i^S and b_{-i}^S must satisfy the following two equalities,

$$\begin{aligned} 0 = \Bigg(&\left(\delta(1-\alpha)^2 - \alpha^2\right)\left(\frac{b_i^S b_{-i}^S}{2} + b_i^S(1 - b_{-i}^S)\right) \\ &+ (1-\alpha)\alpha(\delta-1)\left(\frac{b_i^S}{2}\right)\Bigg) \end{aligned} \tag{A16}$$

$$\begin{aligned} 0 = \Bigg(&\left(\delta(1-\alpha)^2 - \alpha^2\right)\left(\frac{b_{-i}^S b_i^S}{2} + b_{-i}^S(1 - b_i^S)\right) \\ &+ (1-\alpha)\alpha(\delta-1)\left(\frac{b_{-i}^S}{2}\right)\Bigg). \end{aligned} \tag{A17}$$

The symmetric solutions to the preceding mutual best responses are,

$$b_i^S = b_{-i}^S = 0, \tag{A18}$$

and

$$b_i^S = b_{-i}^S = \frac{(2-\alpha)(1-\alpha)\delta - \alpha(1+\alpha)}{(2-\alpha)(1-\alpha)\delta - \alpha(1+\alpha) + \alpha(1+\delta) - \delta}. \tag{A19}$$

The internal solution for b_i^S and b_{-i}^S in (A19) is weakly less than one so long as,

$$\alpha(1+\delta) - \delta \geq 0, \tag{A20}$$

or written differently,

$$\delta \leq \frac{\alpha}{(1-\alpha)}, \tag{A21}$$

and is weakly positive so long that

$$(2-\alpha)(1-\alpha)\delta - \alpha(1+\alpha) \geq 0, \tag{A22}$$

or written differently,

$$\delta \geq \frac{\alpha(1+\alpha)}{(2-\alpha)(1-\alpha)}. \tag{A23}$$

However,

$$\frac{\alpha(1+\alpha)}{(2-\alpha)(1-\alpha)} \geq \frac{\alpha}{1-\alpha}, \tag{A24}$$

because $\alpha > 1/2$. Thus, investors that observe $\hat{B}$ will never invest if $\delta < \frac{\alpha}{1-\alpha}$, i.e., $b_i^S = b_{-i}^S = 0$ for $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha}{1-\alpha}\right)$.

Continuing with $\delta < \frac{\alpha}{1-\alpha}$, consider g_i^S and g_{-i}^S. In this parameter range, $b_i^S = b_{-i}^S = 0$ and investors who observe $\hat{G}$ will invest with probability one, i.e., $g_i^S = g_{-i}^S = 1$ if

$$0 \leq \left(\delta\alpha^2 - (1-\alpha)^2\right)\left(\frac{1}{2}\right) + \alpha(1-\alpha)(\delta - 1). \tag{A25}$$

The threshold value of δ at which the inequality binds is given by,

$$\delta = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right). \tag{A26}$$

Note,

$$\frac{\alpha}{1-\alpha} - \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right) = \frac{\alpha^2(2-\alpha) - (1-\alpha)^2(1+\alpha)}{(1-\alpha)(2-\alpha)\alpha}, \tag{A27}$$

which is weakly positive so long as $\alpha \geq 1/2$.[25] Therefore, $g_i^S = g_{-i}^S = 1$ for $\delta \in \left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right), \frac{\alpha}{1-\alpha}\right)$.

[25] The roots that set the numerator to zero are $\alpha = \frac{1}{2}$, $\alpha = \frac{1}{2}(1-\sqrt{5})$, and $\alpha = \frac{1}{2}(1+\sqrt{5})$. Thus, the only relevant

If $\delta < \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right)$, the mutual best responses must satisfy the following two equalities,

$$0 = \left(\delta\alpha^2 - (1-\alpha)^2\right)\left(\frac{g_i^S g_{-i}^S}{2} + g_i(1-g_{-i})\right) + \alpha(1-\alpha)(\delta-1)g_i^S \tag{A28}$$

$$0 = \left(\delta\alpha^2 - (1-\alpha)^2\right)\left(\frac{g_{-i}^S g_i^S}{2} + g_{-i}(1-g_i)\right) + \alpha(1-\alpha)(\delta-1)g_{-i}^S. \tag{A29}$$

The symmetric solutions to the preceding mutual best responses are,

$$g_i^S = g_{-i}^S = 0, \tag{A30}$$

and

$$g_i^S = g_{-i}^S = \frac{2(\alpha(1+\delta)-1)}{2(\alpha(1+\delta)-1) + (1-\alpha^2(1-\delta)-2\alpha\delta)}. \tag{A31}$$

Focusing on the non-zero solution, the internal mixed strategy is weakly less than one so long as

$$(1-\alpha^2(1-\delta)-2\alpha\delta) \geq 0, \tag{A32}$$

or written differently,

$$\delta \leq \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right). \tag{A33}$$

Furthermore, the internal mixed strategy is weakly greater than zero so long as,

$$2(\alpha(1+\delta)-1) \geq 0, \tag{A34}$$

or written differently,

$$\delta \geq \frac{1-\alpha}{\alpha}. \tag{A35}$$

Therefore, $g_i^S = g_{-i}^S = \frac{2(\alpha(1+\delta)-1)}{2(\alpha(1+\delta)-1)+(1-\alpha^2(1-\delta)-2\alpha\delta)}$ for $\delta \in \left[\frac{1-\alpha}{\alpha}, \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{2-\alpha}\right)\right)$ and $g_i^S = g_{-i}^S = 0$ for $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right)$.

■

Proof of Lemma 2: The proof is constructed as follows: first, we solve for the region of δ for which investors are willing to invest regardless of their signal. Second, we solve for the strategy of investors observing $\hat{B}$ for values of δ smaller than those for which investors invest regardless of their signal. Finally, we solve for the strategy of investors observing $\hat{G}$ for values of δ smaller than

root in $[1/2, 1]$ is $\alpha = 1/2$.

those for which investors invest regardless of their signal, controlling for the strategy of investors that observe $\hat{B}$.

First, consider the case in which investors always invest, regardless of signal, i.e., $\vec{\pi}_i^J = \{1,1\}$ and $\vec{\pi}_{-i}^J = \{1,1\}$. For the equilibrium to exist, it must be the case that $\Pi(\hat{B}|\vec{\pi}_i^J, \vec{\pi}_{-i}^J) \geq 0$. The inequality $\Pi(\hat{B}|\vec{\pi}_i^J, \vec{\pi}_{-i}^J) \geq 0$ binds at,

$$0 = (\delta(1-\alpha)^2 - \alpha^2) + (1-\alpha)\alpha(\delta - 1), \tag{A36}$$

which maps to the same threshold in (A15). Therefore, $b_i^J = b_{-i}^J = g_i^J = g_{-i}^J = 1$ for $\delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right)$.

Now consider $\delta < \frac{\alpha}{1-\alpha}$. Investor i's probability of investing based on a signal $\hat{B}$ must be less than one. By Lemmas A1 and A2, if b^J is an internal value it must be the case that g^J is equal to one. Here, because the outside option is zero, the mutual best responses must satisfy the following two equalities.

$$0 =(\delta(1-\alpha)^2 - \alpha^2)b_i^J b_{-i}^J + (1-\alpha)\alpha(\delta-1)b_i^J \tag{A37}$$

$$0 =(\delta(1-\alpha)^2 - \alpha^2)b_{-i}^J b_i^J + (1-\alpha)\alpha(\delta-1)b_{-i}^J. \tag{A38}$$

The symmetric solutions to the preceding mutual best responses are,

$$b_i^J = b_{-i}^J = 0, \tag{A39}$$

and

$$b_i^J = b_{-i}^J = \frac{(1-\alpha)\alpha(\delta-1)}{(1-\alpha)\alpha(\delta-1) + \alpha(1+\delta) - \delta}. \tag{A40}$$

Focusing on the non-zero solution to the mutual best responses, the expression in (A40) is weakly less than one so long as,

$$\alpha(1+\delta) - \delta \geq 0, \tag{A41}$$

or written differently,

$$\delta \leq \frac{\alpha}{1-\alpha}, \tag{A42}$$

and is weakly positive so long that,

$$(1-\alpha)\alpha(\delta-1) \geq 0, \tag{A43}$$

or written differently,

$$\delta \geq 1. \tag{A44}$$

Therefore, $b_i^J = b_{-i}^J = \frac{(1-\alpha)\alpha(\delta-1)}{(1-\alpha)\alpha(\delta-1)+\alpha(1+\delta)-\delta}$ for $\delta \in \left[1, \frac{\alpha}{1-\alpha}\right)$ and, by Lemma A2, $g_i^J = g_{-i} = 1$ for $\delta \in \left[1, \frac{\alpha}{1-\alpha}\right)$. Also, $b_i^J = b_{-i}^J = 0$ for $\delta \in \left[\frac{(1-\alpha)^2}{\alpha}, 1\right)$.

Now, consider $\delta < 1$ in which $b_i^J = b_{-i}^J = 0$. Investors that observe $\hat{G}$ will invest with probability one, i.e., $g_i^J = g_{-i}^J = 1$, if the following inequality holds,

$$0 \leq \delta\alpha^2 - (1-\alpha)^2. \tag{A45}$$

The preceding inequality binds at,

$$\delta = \left(\frac{1-\alpha}{\alpha}\right)^2. \tag{A46}$$

Therefore, $g_i^J = g_{-i}^J = 1$ for $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right)$.

■

Lemma A3. *The first-best investment criteria for the informed monopolist is given by,*

$$\pi_{\hat{G}\hat{G}}^{FB} = \begin{cases} 1 & \text{if } \delta \geq \frac{(1-\alpha)^2}{\alpha^2} \\ 0 & \text{otherwise.} \end{cases} \tag{A47}$$

$$\pi_{\hat{G}\hat{B}}^{FB} = \begin{cases} 1 & \text{if } \delta \geq 1 \\ 0 & \text{otherwise,} \end{cases} \tag{A48}$$

$$\pi_{\hat{B}\hat{B}}^{FB} = \begin{cases} 1 & \text{if } \delta \geq \frac{\alpha^2}{(1-\alpha^2)} \\ 0 & \text{otherwise.} \end{cases} \tag{A49}$$

Proof of Lemma A3: The first-best investment criteria for the informed monopolist requires the calculation of the monopolist's posterior probabilities based on her two conditionally i.i.d. signals,

$$\Pr(G|\hat{G},\hat{G}) = \frac{\Pr(G\cap\hat{G},\hat{G})}{\Pr(\hat{G},\hat{G})} \tag{A50}$$
$$= \frac{\alpha^2}{\alpha^2+(1-\alpha)^2}, \tag{A51}$$
$$\Pr(G|\hat{G},\hat{B}) = \frac{\Pr(G\cap\hat{G},\hat{B})}{\Pr(\hat{G},\hat{B})} \tag{A52}$$
$$= \frac{\alpha(1-\alpha)}{\alpha(1-\alpha)+\alpha(1-\alpha)} \tag{A53}$$
$$= \frac{1}{2}, \tag{A54}$$
$$\Pr(G|\hat{B},\hat{B}) = \frac{\Pr(G\cap\hat{B},\hat{B})}{\Pr(\hat{B},\hat{B})} \tag{A55}$$
$$= \frac{(1-\alpha)^2}{\alpha^2+(1-\alpha)^2}. \tag{A56}$$

The informed monopolist that observes $\hat{G}$ and $\hat{G}$ will invest as long as she expects a weakly positive return. The return binds at zero at,

$$0 = \frac{\alpha^2}{\alpha^2+(1-\alpha)^2}\delta - \left(1-\frac{\alpha^2}{\alpha^2+(1-\alpha)^2}\right), \tag{A57}$$

implying that $\pi^{FB}_{\hat{G}\hat{G}} = 1$ if δ is sufficiently large,

$$\delta \geq \frac{(1-\alpha)^2}{\alpha^2}. \tag{A58}$$

It is straightforward to solve for the return thresholds for $\pi_{\hat{G}\hat{B}}$ and $\pi_{\hat{B}\hat{B}}$, which are omitted for the sake of brevity.

■

Lemma A4. *The solo value-add is given by,*

$$v_{2,1} = \begin{cases} \frac{\delta-1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \alpha\delta - \frac{1+\alpha^2(\delta-1)}{2} & \delta \in \left[\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}, \frac{\alpha}{1-\alpha}\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}\right). \end{cases} \tag{A59}$$

The joint value-add is given by,

$$v_{2,2} = \begin{cases} \frac{\delta-1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \frac{(2\alpha-1)^2\delta}{2\alpha^2-2(1-\alpha)^2\delta} & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ \alpha - \frac{1+\alpha^2(1-\delta)}{2} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A60}$$

Proof of Lemma A4:

The value-add metric requires the ex ante probability of investment for the solo setting, the joint setting, the uninformed monopolist and the informed monopolist. The ex ante probability of investment is calculated as,

$$\Pr(\text{Funded}) = \Pr(\text{Funded}|G)\Pr(G) + \Pr(\text{Funded}|B)\Pr(B) \tag{A61}$$

$$= \frac{1}{2}\left(\Pr(\text{Funded}|G) + \Pr(\text{Funded}|B)\right). \tag{A62}$$

The value-add metric also requires the ex post probability that a funded project is type G. The conditional probability that a funded project is type G is given by,

$$\Pr(G|\text{Funded}) = \frac{\Pr(G \cap \text{Funded})}{\Pr(\text{Funded})}. \tag{A63}$$

First, consider the solo setting. The ex ante probability that the project is funded is given by,

$$\Pr(\text{Funded}|S) = \frac{1}{2}\left((1 - \Pr(\text{Neither Invests}|G)) + (1 - \Pr(\text{Neither Invests}|B))\right) \tag{A64}$$

$$= 1 - \frac{1}{2}\Big((\alpha^2 + (1-\alpha)^2)((1-g^S)^2 + (1-b^S)^2) + 4\alpha(1-\alpha)(1-g^S)(1-b^S)\Big), \tag{A65}$$

in which S indicates the solo setting, and its explicit solution is,

$$\Pr(\text{Funded}|S) = \begin{cases} 1 & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ 1 - \frac{1}{2}(\alpha^2 + (1-\alpha)^2) & \delta \in \left[\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}, \frac{\alpha}{1-\alpha}\right) \\ \frac{2\alpha(1-\alpha)(2\alpha-1)(1+\delta)(\alpha(1+\delta)-1)}{(1-\alpha(2+\alpha(\delta-1)))^2} & \delta \in \left[\frac{1-\alpha}{\alpha}, \frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right). \end{cases} \tag{A66}$$

The conditional probability that a funded solo project is type G is given by,

$$\Pr(G|\text{Funded}, S) = \frac{\frac{1}{2}\left(1-\alpha^2(1-g^S)^2-(1-\alpha)^2(1-b^S)^2-2\alpha(1-\alpha)(1-g^S)(1-b^S)\right)}{1-\frac{1}{2}\left((\alpha^2+(1-\alpha)^2)((1-g^S)^2+(1-b^S)^2)+4\alpha(1-\alpha)(1-g^S)(1-b^S)\right)}, \tag{A67}$$

and its explicit solution is,

$$\Pr(G|\text{Funded}, S) = \begin{cases} \frac{1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \frac{\alpha(2-\alpha)}{1+2(1-\alpha)\alpha} & \delta \in \left[\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}, \frac{\alpha}{1-\alpha}\right) \\ \frac{1}{1+\delta} & \delta \in \left[\frac{1-\alpha}{\alpha}, \frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}\right) \\ \text{NA} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right). \end{cases} \tag{A68}$$

The preceding probabilities for the solo setting may be combined to calculate the value-add measure,

$$v_{2,1} = \begin{cases} \frac{\delta-1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \alpha\delta - \frac{1+\alpha^2(\delta-1)}{2} & \delta \in \left[\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}, \frac{\alpha}{1-\alpha}\right) \\ 0 & \delta \in \left[\frac{1-\alpha}{\alpha}, \frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right). \end{cases} \tag{A69}$$

Next, consider the joint setting. The ex ante probability that a project is funded is given by,

$$\Pr(\text{Funded}|J) = \frac{1}{2}\left(\Pr(\text{Both Invest}|G) + \Pr(\text{Both Invest}|B)\right) \tag{A70}$$

$$= \frac{1}{2}\left((\alpha^2+(1-\alpha)^2)({g^J}^2+{b^J}^2)+4\alpha(1-\alpha)g^Jb^J\right), \tag{A71}$$

in which J indicates the joint setting, and its explicit solution is,

$$\Pr(\text{Funded}|J) = \begin{cases} 1 & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \frac{(2\alpha-1)^2(\alpha^2+(1-\alpha)^2\delta^2)}{2(\alpha^2(\delta-1)-\delta(2\alpha-1))^2} & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ \frac{1}{2}(\alpha^2+(1-\alpha)^2) & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A72}$$

The conditional probability that a funded joint project is type G is given by,

$$\Pr(G|\text{Funded}, J) = \frac{\alpha^2 {g^J}^2 + (1-\alpha)^2 {b^J}^2 + 2\alpha(1-\alpha) g^J b^J}{(\alpha^2 + (1-\alpha)^2)({g^J}^2 + {b^J}^2) + 4\alpha(1-\alpha) g^J b^J} \tag{A73}$$

and its explicit solution is,

$$\Pr(G|\text{Funded}, J) = \begin{cases} \frac{1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \frac{\alpha^2}{\alpha^2+(1-\alpha)^2\delta^2} & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ \frac{\alpha^2}{1-2(1-\alpha)\alpha} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A74}$$

The preceding probabilities for the joint setting may be combined to calculate the value-add measure,

$$v_{2,2} = \begin{cases} \frac{\delta-1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \frac{(2\alpha-1)^2\delta}{2\alpha^2-2(1-\alpha)^2\delta} & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ \alpha - \frac{1+\alpha^2(1-\delta)}{2} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A75}$$

■

Lemma A5. *The first-best value-add is given by,*

$$v_2^{FB} = \begin{cases} \frac{\delta-1}{2} & \delta \in \left[\frac{\alpha^2}{(1-\alpha)^2}, \infty\right) \\ \alpha\delta - \frac{1+\alpha^2(\delta-1)}{2} & \delta \in \left[1, \frac{\alpha^2}{(1-\alpha)^2}\right) \\ \alpha - \frac{1+\alpha^2(1-\delta)}{2} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A76}$$

Proof of Lemma A5:

Consider the first-best value-add from an informed monopolist. The ex ante probability that the informed monopolist funds a project is,

$$\Pr(\text{Funded}|FB) = \frac{1}{2}\left(\Pr(\text{Invest}|G) + \Pr(\text{Invest}|B)\right) \tag{A77}$$

$$= \frac{1}{2}\left((\alpha^2 + (1-\alpha)^2)\pi_{GG} + 4\alpha(1-\alpha)\pi_{GB} + (\alpha^2 + (1-\alpha)^2)\pi_{BB}\right). \tag{A78}$$

in which FB indicates the first-best setting, and its explicit solution is,

$$\Pr(\text{Funded}|FB) = \begin{cases} 1 & \delta \in \left[\frac{\alpha^2}{(1-\alpha)^2}, \infty\right) \\ \frac{1}{2}\left((\alpha^2 + (1-\alpha)^2) + 4\alpha(1-\alpha)\right) & \delta \in \left[1, \frac{\alpha^2}{(1-\alpha)^2}\right) \\ \frac{1}{2}(\alpha^2 + (1-\alpha)^2) & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A79}$$

The conditional probability that a project funded by the informed monopolist is type G is given by,

$$\Pr(G|\text{Funded}, FB) = \begin{cases} \frac{1}{2} & \delta \in \left[\frac{\alpha^2}{(1-\alpha)^2}, \infty\right) \\ \frac{\alpha^2+2\alpha(1-\alpha)}{\alpha^2+(1-\alpha)^2+4\alpha(1-\alpha)} & \delta \in \left[1, \frac{\alpha^2}{(1-\alpha)^2}\right) \\ \frac{\alpha^2}{\alpha^2+(1-\alpha)^2} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A80}$$

The preceding probabilities for the informed monopolist setting may be combined to calculate the value-add measure,

$$v_2^{FB} = \begin{cases} \frac{\delta-1}{2} & \delta \in \left[\frac{\alpha^2}{(1-\alpha)^2}, \infty\right) \\ \alpha\delta - \frac{1+\alpha^2(\delta-1)}{2} & \delta \in \left[1, \frac{\alpha^2}{(1-\alpha)^2}\right) \\ \alpha - \frac{1+\alpha^2(1-\delta)}{2} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A81}$$

■

Proof of Proposition 1: The explicit forms of the value-adds in (A69), (A75), and (A81) are utilized in the proof of this proposition. The difference between the first-best value-add measure and the solo setting value-add is given by,[26]

$$v_2^{FB} - v_{2,1} = \begin{cases} \frac{\alpha^2-(1-\alpha)^2\delta}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \frac{\alpha^2}{(1-\alpha)^2}\right) \\ 0 & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ \alpha(1-\alpha)(1-\delta) & \delta \in \left[\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}, 1\right) \\ \alpha - \frac{1+\alpha^2(1-\delta)}{2} & \delta \in \left[\frac{1-\alpha}{\alpha}, \frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}\right) \\ \alpha - \frac{1+\alpha^2(1-\delta)}{2} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right). \end{cases} \tag{A82}$$

The difference between the value-add measures in the informed monopolist setting and the joint

[26]Note, $\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}$ is strictly smaller than one for all values of $\alpha \in \left(\frac{1}{2}, 1\right]$.

setting is given by,

$$v_2^{FB} - v_{2,2} = \begin{cases} \alpha\delta - \frac{1+\alpha^2(\delta-1)}{2} - \frac{\delta-1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \frac{\alpha^2}{(1-\alpha)^2}\right) \\ \alpha\delta - \frac{1+\alpha^2(\delta-1)}{2} - \frac{(2\alpha-1)^2\delta}{2\alpha^2-2(1-\alpha)^2\delta} & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A83}$$

■

Lemma A6. *The second-best value-add measures for the solo and joint settings are given by,*

$$v_{2,1}^{SB} = \begin{cases} \alpha\delta - \frac{1+\alpha^2(\delta-1)}{2} & \delta \in \left[1, \frac{\alpha^2}{(1-\alpha)^2}\right] \\ \frac{(1-\alpha(1+\delta))^2}{4\alpha-2-2\alpha^2(1-\delta)} & \delta \in \left[\frac{1-\alpha}{\alpha}, 1\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right), \end{cases} \tag{A84}$$

and

$$v_{2,2}^{SB} = \begin{cases} \frac{\delta-1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \frac{(2\alpha-1)^2\delta}{2\alpha^2-2(1-\alpha)^2\delta} & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ \alpha - \frac{1+\alpha^2(1-\delta)}{2} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A85}$$

Proof of Lemma A6: The second-best value-add is the maximum joint surplus that can be achieved when investors cannot coordinate their signals. As such, it is equivalent to a setting in which a social planner dictates a strategy set $\{g, b\}$ that maximizes joint surplus. Beginning with the solo setting in which there are $N = 2$ investors and only $n = 1$ are required to fund the project, the planner's problem is formally given by,

$$\max_{g,b\in[0,1]} v_{2,1}(g, b), \tag{A86}$$

where,

$$v_{2,1}(g, b) = \Pr(\text{Funded}|g, b, S)\big(\Pr(G|g, b, \text{Funded}, S)\delta - \Pr(B|g, b, \text{Funded}, S)\big). \tag{A87}$$

Equations (A65) and (A67) give the explicit form of $v_{2,1}(g, b)$,

$$\begin{aligned} v_{2,1}(g, b) =& \frac{1}{2}\left(1 - \alpha^2(1-g)^2 - (1-\alpha)^2(1-b)^2 - 2\alpha(1-\alpha)(1-g)(1-b)\right)(\delta+1) \\ &- \left(1 - \frac{1}{2}\left((\alpha^2 + (1-\alpha)^2)((1-g)^2 + (1-b)^2) + 4\alpha(1-\alpha)(1-g)(1-b)\right)\right). \end{aligned} \tag{A88}$$

We begin by considering the optimal strategy for an investor that observes $\hat{B}$. Note that $v_{2,1}(g,b)$ is convex in b,

$$\frac{\mathrm{d}^2}{\mathrm{d}b^2}\left(v_{2,1}(g,b)\right) = \alpha^2 - (1-\alpha)^2\delta, \tag{A89}$$

because the preceding expression is positive for all values of $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right]$. Furthermore, the first derivative with respect to $\hat{B}$,

$$\frac{\mathrm{d}}{\mathrm{d}b}\left(v_{2,1}(1,b) = (-\alpha^2(1-\delta) + \delta(1-2\alpha))(1-b),\right. \tag{A90}$$

is negative for all values of $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right]$, i.e., value-add is decreasing in b. Therefore, the optimal strategy for investors observing $\hat{B}$ in the solo setting is,

$$b_{2,1}^{SB} = \begin{cases} 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right] \end{cases} \tag{A91}$$

Now consider the optimal strategy for an investor that observes $\hat{G}$. Note that $v_{2,1}(g,b)$ is concave in g,

$$\frac{\mathrm{d}^2}{\mathrm{d}g^2}\left(v_{2,1}(g,b)\right) = 1 + \alpha(-2 + \alpha(1-\delta)), \tag{A92}$$

because the preceding expression is negative for all values of $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right]$. The joint payoff is maximized with the strategy,

$$g_{2,1}^{SB} = \begin{cases} 1 & \delta \in \left[1, \frac{\alpha^2}{(1-\alpha)^2}\right] \\ \frac{1-\alpha(1+\delta)}{1-\alpha(1+\delta)-\alpha(1-\alpha)(1-\delta)} & \delta \in \left[\frac{(1-\alpha)}{\alpha}, 1\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{(1-\alpha)}{\alpha}\right) . \end{cases} \tag{A93}$$

Using the explicit strategies $g_{2,1}^{SB}$ and $b_{2,1}^{SB}$, the second-best value-add is given by,

$$v_{2,1}^{SB} = \begin{cases} \alpha\delta - \frac{1+\alpha^2(\delta-1)}{2} & \delta \in \left[1, \frac{\alpha^2}{(1-\alpha)^2}\right] \\ \frac{(1-\alpha(1+\delta))^2}{4\alpha-2-2\alpha^2(1-\delta)} & \delta \in \left[\frac{1-\alpha}{\alpha}, 1\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right) . \end{cases} \tag{A94}$$

Continuing with the joint setting in which there are $N = 2$ investors and $n = 2$ are required to fund the project, the planner's problem is formally given by,

$$\max_{g,b\in[0,1]} v_{2,2}(g,b), \tag{A95}$$

where,

$$v_{2,2}(g,b) = \Pr(\text{Funded}|g,b,J)\big(\Pr(G|g,b,\text{Funded},J)\delta - \Pr(B|g,b,\text{Funded},J)\big). \tag{A96}$$

Equations (A71) and (A73) give the explicit form of $v_{2,2}(g,b)$,

$$\begin{aligned} v_{2,2}(g,b) = &\left(\alpha^2 g^2 + (1-\alpha)^2 b^2 + 2\alpha(1-\alpha)gb\right)(\delta+1) \\ &- \left((\alpha^2 + (1-\alpha)^2)(g^2+b^2) + 4\alpha(1-\alpha)gb\right). \end{aligned} \tag{A97}$$

We begin by considering the optimal strategy for an investor that observes $\hat{G}$. Note that $v_{2,2}(g,b)$ is concave in b,

$$\frac{\mathrm{d}^2}{\mathrm{d}b^2}\left(v_{2,2}(g,b)\right) = -2\alpha^2 + 2(1-\alpha)^2\delta, \tag{A98}$$

because the preceding expression is negative for all values of $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right]$. Therefore, when $g = 1$ the mixing strategy that maximizes joint surplus for investors observing $\hat{B}$ is,

$$b_{2,2} = \begin{cases} 1 & \delta \in \left[\frac{\alpha}{1-\alpha}, \frac{\alpha^2}{(1-\alpha)^2}\right] \\ \frac{(1-\alpha)\alpha(\delta-1)}{(1-\alpha)\alpha(\delta-1)+\alpha(1+\delta)-\delta} & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right). \end{cases} \tag{A99}$$

Note that $v_{2,2}(g,b)$ is convex in g,

$$\frac{\mathrm{d}^2}{\mathrm{d}g^2}\left(v_{2,2}(g,b)\right) = 2\alpha(2+\alpha(\delta-1)) - 2, \tag{A100}$$

because the preceding expression is positive for all values of $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right]$. Furthermore, the first derivative with respect to b,

$$\frac{\mathrm{d}}{\mathrm{d}g}\left(v_{2,2}(g,b) = (2(-1+\alpha(2+\alpha(\delta-1)))g\right. \tag{A101}$$

is positive for all values of $\delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right]$, i.e., value-add is increasing in g. Therefore, the optimal strategy for investors observing $\hat{G}$ in the joint setting is,

$$g_{2,2}^{SB} = \begin{cases} 1 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right] . \end{cases} \tag{A102}$$

Using the explicit strategies $g_{2,2}^{SB}$ and $b_{2,2}^{SB}$, the second-best value-add is given by,

$$v_{2,2}^{SB} = \begin{cases} \frac{\delta-1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \frac{(2\alpha-1)^2\delta}{2\alpha^2-2(1-\alpha)^2\delta} & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ \alpha - \frac{1+\alpha^2(1-\delta)}{2} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, 1\right) . \end{cases} \tag{A103}$$

■

Proof of Corollary 1.1:

The explicit forms of the value-adds in (A69), (A75), (A94), and (A103) are utilized in the proof of this corollary. The difference between the second-best value-add measure and the solo setting value-add is given by,

$$v_{2,1}^{SB} - v_{2,1} = \begin{cases} \frac{(\alpha^2-(1-\alpha)^2\delta)}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \frac{\alpha^2}{(1-\alpha)^2}\right] \\ 0 & \delta \in \left[1, \frac{\alpha}{1-\alpha}\right) \\ \frac{(1-\alpha)^2\alpha^2(1-\delta)^2}{4\alpha-2-2\alpha^2(1-\delta)} & \delta \in \left[\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}, 1\right) \\ \frac{(1-\alpha(1+\delta))^2}{4\alpha-2-2\alpha^2(1-\delta)} & \delta \in \left[\frac{1-\alpha}{\alpha}, \frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right) . \end{cases} \tag{A104}$$

The difference between the second-best value-add measure and the join setting value-add is given by,

$$v_{2,2}^{SB} - v_{2,2} = \begin{cases} 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{\alpha^2}{(1-\alpha)^2}\right] . \end{cases} \tag{A105}$$

■

Lemma A7. *If there was only one investor, receiving a single signal, her investment criteria is* *given by,*

$$\pi^{SI}_{\hat{G}} = \begin{cases} 1 & \textit{if } \delta \geq \frac{1-\alpha}{\alpha} \\ 0 & \textit{otherwise.} \end{cases} \tag{A106}$$

$$\pi^{SI}_{\hat{B}} = \begin{cases} 1 & \textit{if } \delta \geq \frac{\alpha}{1-\alpha} \\ 0 & \textit{otherwise.} \end{cases} \tag{A107}$$

Proof of Lemma A7:

The investment criteria is straightforward and omitted for the sake of brevity.

■

Proof of Corollary 1.2:

The single investor's investment criteria is outlined in Lemma A7. The single investor's value-add is now calculated. The ex ante probability that the project is funded is,

$$\Pr(F|SI) = \frac{1}{2}\left(\Pr(\text{Invest}|G) + \Pr(\text{Invest}|B)\right) \tag{A108}$$

$$= \frac{1}{2}\left(g + b\right), \tag{A109}$$

in which SI indicates the single-investor setting, and its explicit solution is,

$$\Pr(F|SI) = \begin{cases} 1 & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \frac{1}{2} & \delta \in \left[\frac{1-\alpha}{\alpha}, \frac{\alpha}{1-\alpha}\right) \\ 0 & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right). \end{cases} \tag{A110}$$

The conditional probability that a project funded by the uninformed monopolist is type G is given by,

$$\Pr(G|F, SI) = \begin{cases} \frac{1}{2} & \delta \in \left[\frac{\alpha}{1-\alpha}, \infty\right) \\ \alpha & \delta \in \left[\frac{1-\alpha}{\alpha}, \frac{\alpha}{1-\alpha}\right) \\ \text{NA} & \delta \in \left[\frac{(1-\alpha)^2}{\alpha^2}, \frac{1-\alpha}{\alpha}\right). \end{cases} \tag{A111}$$

The preceding probabilities for the uninformed monopolist setting may be combined to calculate the value-add measure,

$$v_{1,1}=\begin{cases}\frac{\delta-1}{2} & \delta\in\left[\frac{\alpha}{1-\alpha},\infty\right)\\ \frac{\alpha(\delta+1)-1}{2} & \delta\in\left[\frac{1-\alpha}{\alpha},\frac{\alpha}{1-\alpha}\right)\\ 0 & \delta\in\left[\frac{(1-\alpha)^2}{\alpha^2},\frac{1-\alpha}{\alpha}\right).\end{cases}\tag{A112}$$

The explicit forms of the value-adds in (A69), (A75), and (A112) are utilized to calculate over and under-performance relative to the single investor. The difference between the value-add measures in the single investor setting and the solo setting is given by,[27]

$$v_{1,1}-v_{2,1}=\begin{cases}0 & \delta\in\left[\frac{\alpha}{1-\alpha},\infty\right)\\ -\frac{(1-\alpha)\alpha(\delta-1)}{2}\leq 0 & \delta\in\left[1,\frac{\alpha}{1-\alpha}\right)\\ \frac{(1-\alpha)\alpha(1-\delta)}{2}>0 & \delta\in\left[\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)},1\right)\\ \frac{\alpha(\delta+1)-1}{2}\geq 0 & \delta\in\left[\frac{1-\alpha}{\alpha},\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}\right)\\ 0 & \delta\in\left[\frac{(1-\alpha)^2}{\alpha^2},\frac{1-\alpha}{\alpha}\right).\end{cases}\tag{A113}$$

The difference between the value-add measures in the informed monopolist setting and the joint setting is given by,

$$v_{1,1}-v_{2,2}=\begin{cases}0 & \delta\in\left[\frac{\alpha}{1-\alpha},\infty\right)\\ \frac{(1-\alpha)\alpha(\delta-1)(\alpha-(1-\alpha)\delta)}{2\alpha^2-2(1-\alpha)^2\delta}\geq 0 & \delta\in\left[1,\frac{\alpha}{1-\alpha}\right)\\ -\frac{(1-\alpha)\alpha(1-\delta)}{2}<0 & \delta\in\left[\frac{1-\alpha}{\alpha},1\right)\\ -\left(\alpha-\frac{1+\alpha^2(1-\delta)}{2}\right)\leq 0 & \delta\in\left[\frac{(1-\alpha)^2}{\alpha^2},\frac{1-\alpha}{\alpha}\right)\end{cases}\tag{A114}$$

■

We establish several useful results before proving Section 3's lemmas and propositions.

Lemma A8. *Conditional on investors' equilibrium participation strategies $\{g^{NP},b^{NP}\}$, an investor uses an exposure weight $\phi(g^{NP},b^{NP}|\hat{F})$ in place of her subjective state probability in choosing whether or not to invest.*

[27]Note, $\frac{(1-\alpha)(1+\alpha)}{\alpha(2-\alpha)}$ is strictly smaller than one for all values of $\alpha\in\left(\frac{1}{2},1\right]$.

Proof of Lemma A8: Investors' expectations of the project's value depend on their signals:

$$E\big[V|\hat{F}\big] = \begin{cases} (\alpha(\delta+1)-1)\,c & \hat{F}=\hat{G} \\ ((1-\alpha)(\delta+1)-1)\,c & \hat{F}=\hat{B}. \end{cases} \tag{A115}$$

Investors do not necessarily invest when $E\big[V|\hat{F}\big] \geq 0$ due to their asymmetric exposure to project outcomes. Recall, the share of M that is actually invested in the project is state dependent. Specifically, in (23), the share of investors' committed capital deployed to the project $\sigma(F)$ and the remaining $1-\sigma(F)$ is returned to investors implying that each investor's investment in the risky project is,

$$\sigma(F)M. \tag{A116}$$

Conditional on the equilibrium participation strategies of investors, an explicit form for σ as a function of F, g^{NP}, and b^{NP} is given by,

$$\sigma(F, g^{NP}, b^{NP}) = \begin{cases} \frac{c\mathbb{1}_G}{M(g^{NP}\alpha+b^{NP}(1-\alpha))} & F=G \\ \frac{c\mathbb{1}_B}{M(g^{NP}(1-\alpha)+b^{NP}\alpha)} & F=B. \end{cases} \tag{A117}$$

where $\mathbb{1}_F$ is shorthand notation for an indicator function detailed hereafter. Define,

$$\mathcal{S}(F, g^{NP}, b^{NP}), \tag{A118}$$

as the set of projects funded in state F. The set $\mathcal{S}(B, g^{NP}, b^{NP})$ contains all projects with scale

$$c \leq M\left(g^{NP}(1-\alpha)+b^{NP}\alpha\right),$$

and $\mathcal{S}(G, g^{NP}, b^{NP})$ contains all projects with scale,

$$c \leq M\left(g^{NP}\alpha+b^{NP}(1-\alpha)\right).$$

Some intuition regarding the importance of these sets is in order. Consider the following example: a project with scale $\hat{c} \notin \mathcal{S}(B, g^{NP}, b^{NP})$ will not be funded if it is not financially sound. Because the project will not attract sufficient capital in the state $F = B$, investors receive an implicit hedge against that state because of the all-or-nothing feature. As such, $\mathbb{1}_F$ is shorthand notation for the indicator function $\mathbb{1}_{c\in\mathcal{S}(F,g^{NP},b^{NP})}$.

Investors will invest in a project if their exposure to the state $F = G$ is weakly greater their investment. Roughly speaking, define an exposure weight for investors as,

$$\phi(g^{NP}, b^{NP}|\hat{F}) = \begin{cases} \frac{\alpha\sigma(G,g^{NP},b^{NP})}{\alpha\sigma(G,g^{NP},b^{NP})+(1-\alpha)\sigma(B,g^{NP},b^{NP})} & \hat{F} = \hat{G}, \\ \frac{(1-\alpha)\sigma(G,g^{NP},b^{NP})}{(1-\alpha)\sigma(G,g^{NP},b^{NP})+\alpha\sigma(B,g^{NP},b^{NP})} & \hat{F} = \hat{B}. \end{cases} \tag{A119}$$

As such, an investor that observes $\hat{F}$ will invest if,

$$\phi(g^{NP}, b^{NP}|\hat{F})(\delta + 1) - 1 \geq 0. \tag{A120}$$

■

Lemma A9. *The exposure weight $\phi(g^{NP}, b^{NP}|\hat{B})$ is weakly increasing in b^{NP}.*

Proof of Lemma A9:

If $\mathbb{1}_B = 0$, the comparative statics of $\phi(g^{NP}, b^{NP}|\hat{B})$ and $\phi(g^{NP}, b^{NP}|\hat{G})$ are trivial: they all equal zero. For the remainder of the proof, assume $\mathbb{1}_B = 1$.

The expression $\phi(g^{NP}, b^{NP}|\hat{B})$ may be rewritten as,

$$\phi(g^{NP}, b^{NP}|\hat{B}) = \frac{1}{1 + \eta_B}, \tag{A121}$$

where,

$$\eta_B = \frac{\alpha \left(g^{NP}\alpha + b^{NP}(1-\alpha)\right)}{(1-\alpha)\left(g^{NP}(1-\alpha) + b^{NP}\alpha\right)}. \tag{A122}$$

Therefore, the comparative static of $\phi(g^{NP}, b^{NP}|\hat{B})$ with respect to b^{NP} is,

$$\frac{\partial\phi(g^{NP}, b^{NP}|\hat{B})}{\partial b^{NP}} = -\frac{\partial\eta_B/\partial b^{NP}}{(1+\eta_B)^2}. \tag{A123}$$

The partial derivative of η_B with respect to b^{NP} is given by,

$$\frac{\partial\eta_B}{\partial b^{NP}} = \frac{\alpha((1-2\alpha)g^{NP})}{(1-\alpha)\left(g^{NP}(1-\alpha) + b^{NP}\alpha\right)^2}. \tag{A124}$$

The sign on the preceding expression is determined by the sign on,

$$(1-2\alpha)g^{NP}, \tag{A125}$$

which is negative because $\alpha > \frac{1}{2}$. Therefore $\frac{\partial \eta_B}{\partial b^{NP}} < 0$ implying that,

$$\frac{\partial \phi(g^{NP}, b^{NP}|\hat{B})}{\partial b^{NP}} \geq 0. \tag{A126}$$

■

Lemma A10. *Any equilibrium featuring a loser's blessing, i.e., the project's scale c is not in the set $\mathcal{S}(F, g^{NP}, b^{NP})$, also features full investor participation.*

Proof of Lemma A10:

An equilibrium featuring a loser's blessing ensures an expected profit with no possible loss. Any non-participating investor would not be acting optimally.

■

Lemma A11. *Investors that observe $\hat{F} = \hat{B}$ adhere to a deterministic strategy, $b^{NP} \in \{0, 1\}$. If there is not a loser's blessing and*

$$\delta < \frac{\alpha}{1-\alpha}, \tag{A127}$$

no investors observing $\hat{F} = \hat{B}$ invest in the project.

Proof of Lemma A11: By Lemma A10 all investors participate if a loser's blessing exists. Consider the case in which a loser's blessing does not exist, i.e., bad projects are funded with probability one. All investors that observe $\hat{F} = \hat{B}$ will invest if and only if,

$$0 \leq (1-\alpha)\left(\frac{(\delta+1)c - c}{c}\right) M\sigma(G, 1, 1) - \alpha\left(\frac{c}{c}\right) M\sigma(B, 1, 1). \tag{A128}$$

The inequality simplifies to,

$$1 \leq (\delta+1)\frac{(1-\alpha)\sigma(G, 1, 1)}{(1-\alpha)\sigma(G, 1, 1) + \alpha\sigma(B, 1, 1)} \equiv (\delta+1)\phi(1, 1|\hat{B}). \tag{A129}$$

If the preceding inequality holds with equality, then for any return marginally smaller than δ it must be the case that not all investors participate. Furthermore, investors that observe $\hat{B}$ cannot employ a mixing strategy because,

$$\phi(1, 1|\hat{B}) > \phi(1, b^{NP}|\hat{B}), \tag{A130}$$

for any $b^{NP} < 1$ by Lemma A9. Therefore, for any $(\delta + 1)$ smaller than $1/\phi(1, 1|B)$, investors that observe $\hat{F} = \hat{B}$ will not participate. Thus, the investors' decision to participate is deterministic when they observe $\hat{F} = \hat{B}$.

The threshold return that compels the set of investors that observe $\hat{F} = \hat{B}$ to invest is implicitly defined by the following equality (using (A117) and (A119)),

$$(\delta + 1)\left(\frac{\frac{c(1-\alpha)}{M}}{\frac{c(1-\alpha)}{M} + \frac{c\alpha}{M}}\right) = 1, \tag{A131}$$

which simplifies to,

$$(\delta + 1)\left(\frac{1}{1 + \frac{\alpha}{1-\alpha}}\right) = 1. \tag{A132}$$

Therefore, for all investors to participate (regardless of signal) the promised return needs to be sufficiently large,

$$\delta \geq \frac{\alpha}{1 - \alpha}. \tag{A133}$$

■

Lemma A12. *If*

$$1 \leq \delta < \frac{\alpha}{1 - \alpha}, \tag{A134}$$

and $c \leq M(1-\alpha)$, then investors that observe $\hat{F} = \hat{B}$ do not participate and a investor that observes $\hat{F} = \hat{G}$ will invest with probability one, $g^{NP} = 1$.

Proof of Lemma A12: By Lemma A10 all investors participate if a loser's blessing exists. Consider the case in which a loser's blessing does not exist (and is not triggered when investors observing $\hat{B}$ do not participate — we consider that possibility in Lemma A14). Suppose $\delta < \frac{\alpha}{1-\alpha}$ so investors' observing $\hat{F} = \hat{B}$ do not participate. Investors that observe $\hat{F} = \hat{G}$ will participate if and only if,

$$0 \leq \alpha\left(\frac{(\delta + 1)c - c}{c}\right) M\sigma(G, 1, 0)\mathbb{1}_G - (1 - \alpha)\left(\frac{c}{c}\right) M\sigma(B, 1, 0)\mathbb{1}_B. \tag{A135}$$

The inequality simplifies to,

$$1 \leq (\delta + 1)\frac{\alpha\sigma(G, 1, 0)}{\alpha\sigma(G, 1, 0) + (1 - \alpha)\sigma(B, 1, 0)\mathbb{1}_B} \equiv (\delta + 1)\frac{1}{2}. \tag{A136}$$

The preceding inequality holds with equality at $\delta = 1$. Furthermore, the non-participation of investors observing $\hat{B}$ does not trigger a loser's blessing as $c \leq M(1-\alpha)$ and even a project of type B is financed with certainty.

■

Lemma A13. *If*

$$\delta < 1 \tag{A137}$$

no investors participate.

Proof of Lemma A13:

The proof comes from Lemma A12.

■

Lemma A14. *If*

$$1 \leq \delta < \frac{\alpha}{1-\alpha}, \tag{A138}$$

and $c > M(1-\alpha)$, then the only equilibrium is one in which no project is financed.

Proof of Lemma A14:

First, consider projects with scale $c \in ((1-\alpha)M, \alpha M]$. According to Lemma A11, investors observing $\hat{B}$ will not participate if $\delta < \frac{\alpha}{1-\alpha}$. And, by Lemma A12, all investors observing $\hat{G}$ will participate if $\delta \in \left[\delta, \frac{\alpha}{1-\alpha}\right)$. Therefore, a good project will attract αM units of capital and a bad project will attract $(1-\alpha)M$ units of capital. However, because financing is all-or-nothing, a bad project will not raise sufficient capital because $c > (1-\alpha)M$ and the project will be canceled, inducing a loser's blessing. By Lemma A10, any equilibrium featuring a loser's blessing includes full investor participation, implying investors observing $\hat{B}$ would be compelled to participate. However, by participating, and because investors observing $\hat{B}$ play deterministic strategies by Lemma A11, participation by investors observing $\hat{B}$ unravels the loser's blessing and those investors will no longer want to participate. As such, no equilibrium exists in which the project is financed and the only equilibrium is one in which no project is financed.

Now, consider projects with scale $c > \alpha M$. In this case, while investors observing $\hat{G}$ would like to participate because $\delta \in \left[1, \frac{\alpha}{1-\alpha}\right)$, they lack sufficient capital to finance the project as they can only raise αM for good projects. As such, no equilibrium exists in which the project is financed and the only equilibrium is one in which no project is financed.

■

Proof of Lemma 3:

If a loser's blessing exists, i.e., $\mathbb{1}_B = 0$, then investors' payoffs from participating are strictly positive. This implies that the unit measure of investors collectively provides M units of capital regardless of the project's type. Thus, if $\mathbb{1}_B = 0$ it must be the case that $\mathbb{1}_G$ is also equal to zero and that the project cannot be financed, i.e., $c > M$. Therefore, a loser's blessing cannot exist in equilibrium.

■

Proof of Lemma 4:

The proof closely resembles the proofs of Lemmas A10-A14 and is omitted for the sake of brevity.

■

Proof of Lemma 5:

The proof closely resembles the proofs of Lemmas A10-A14 and is omitted for the sake of brevity.

■

Proof of Proposition 2:

The proof is derived from the results of Lemma 4 and Lemma 5.

■

Appendix B

Equations (20) and (21) give investors' conditions for contributing capital to a project. In equilibrium, all investors' strategies must satisfy these conditions. To solve for the equilibrium, we begin by formally defining the probabilities used in (20) and (21). Taking all investors' strategies as given, the probability of realizing state n if the project is good is (we suppress the superscripts LC for the sake of exposition throughout this appendix):

$$\Pr(n|G,\vec{\pi},N,\alpha) = \sum_{y=0}^{N} \sum_{x=\max(0,n-(N-y))}^{\min(y,n)} \left(\alpha^{y} \prod_{j\in X} g_j \prod_{j\in Y, j\notin X} (1-g_j)\right) \times \left((1-\alpha)^{N-y} \prod_{j\in K, j\notin Y} b_j \prod_{j\notin K, j\notin Y} (1-b_j)\right) \binom{N}{y}\binom{y}{x}\binom{N-y}{n-x}, \quad \text{(B1)}$$

where K is the set of n investors who participate, Y is the set of investors who receive good signals and X is the intersection of K and Y. In the summations, y indexes the size of Y and x indexes the size of X. The three counting functions provide the number of unique ways one can select y investors (out of N) who receive good signals, x investors (out of y) who receive good signals and contribute, and $n-x$ investors (out of the $N-y$ investors who receive bad signals) who receive bad signals and contribute. The probability of realizing state n if the project is bad is:

$$\Pr(n|B,\vec{\pi},N,\alpha) = \sum_{y=0}^{N} \sum_{x=\max(0,n-(N-y))}^{\min(y,n-1)} \left((1-\alpha)^{y} \prod_{j\in X,} g_j \prod_{j\in Y, j\notin X} (1-g_j)\right) \times \left(\alpha^{N-y} \prod_{j\in K, j\notin Y} b_j \prod_{j\notin K, j\notin Y} (1-b_j)\right) \binom{N}{y}\binom{y}{x}\binom{N-y}{n-x}. \quad \text{(B2)}$$

(B1) and (B2) only differ in the probabilities applied to investors' receiving good or bad signals. For good projects, investors are more likely to receive good signals (with probability α), while for bad projects, investors are more likely to receive bad signals (also with probability α).

To calculate the probabilities for an individual investor (which are used in (20) and (21)), other investors' strategies, $\vec{\pi}_{-i}$, are used in place of all investors' strategies, $\vec{\pi}$, $N-1$ is used in place of

N, and the sets K, Y and X exclude investor i. As a result, (B1) becomes:

$$\Pr_i(n|G, \vec{\pi}_{-i}, N-1, \alpha) = \sum_{y=0}^{N-1} \sum_{x=\max(0,n-(N-1-y))}^{\min(y,n)} \left(\alpha^y \prod_{j \in X, j \neq i} g_j \prod_{j \in Y, j \notin X, j \neq i} (1-g_j) \right) \times \left((1-\alpha)^{N-1-y} \prod_{j \in K, j \notin Y, j \neq i} b_j \prod_{j \notin K, j \notin Y, j \neq i} (1-b_j) \right) \times \binom{N-1}{y} \binom{y}{x} \binom{N-1-y}{n-x}, \tag{B3}$$

and (B2) becomes:

$$\Pr_i(n|B, \vec{\pi}_{-i}, N-1, \alpha) = \sum_{y=0}^{N-1} \sum_{x=\max(0,n-(N-1-y))}^{\min(y,n)} \left((1-\alpha)^y \prod_{j \in X, j \neq i} g_j \prod_{j \in Y, j \notin X, j \neq i} (1-g_j) \right) \times \left(\alpha^{N-1-y} \prod_{j \in K, j \notin Y, j \neq i} b_j \prod_{j \notin K, j \notin Y, j \neq i} (1-b_j) \right) \times \binom{N-1}{y} \binom{y}{x} \binom{N-1-y}{n-x}. \tag{B4}$$

To solve for the equilibrium strategies, we first develop two lemmas.

Lemma B1. *For any internal mixing probability $\pi_{\hat{F},i} \in (0,1)$ to be an equilibrium strategy, $\Pi_i(\hat{F}) = 0$. If $\pi_{\hat{F},i} = 0$, then $\Pi_i(\hat{F}) \leq 0$. If $\pi_{\hat{F},i} = 1$, then $\Pi_i(\hat{F}) \geq 0$.*

Proof of Lemma B1:

Suppose not. Begin with the case in which $\pi_{\hat{F},i} \in (0,1)$ and $\Pi_i(\hat{F}) > 0$. Because the investor will always contribute after observing $\hat{F}$, the investor will deviate whenever the mixing probability dictates not contributing. Similarly, if $\pi_{\hat{F},i} \in (0,1)$ and $\Pi_i(\hat{F}) < 0$ the investor will never contribute after observing $\hat{F}$, deviating whenever the mixing probability dictates to contribute. Next, consider the case when $\pi_{\hat{F},i} = 0$ and $\Pi_i(\hat{F}) > 0$. As before, the investor will always contribute after observing $\hat{F}$, and thus will always deviate from the strategy of not contributing. Similarly, if $\pi_{\hat{F},i} = 1$ and $\Pi_i(\hat{F}) < 0$ the investor will never contribute after observing $\hat{F}$, and thus will always deviate from the strategy of always contributing.

■

Lemma B1 follows from investors' playing incentive compatible strategies. Because investors will always invest if $\Pi_i(\hat{F}) > 0$ and will never invest when $\Pi_i(\hat{F}) < 0$, these behaviors can only be supported in equilibrium by the strategies $\pi_{\hat{F},i} = 1$ and $\pi_{\hat{F},i} = 0$, respectively. Similarly, to support mixing in equilibrium, investors must be indifferent between investing and not investing, meaning $\Pi_i(\hat{F}) = 0$. The requirement that strategies be incentive compatible leads to the following lemma.

Lemma B2. *In any equilibrium of the financing game with a discrete number of investors, all agents play identical strategies conditional on their signals, i.e. $b_i = b_j$ and $g_i = g_j$ $\forall$ i, j.*

Proof of Lemma B2:

We first show that only one of (20) and (21) can hold with equality. Suppose not. Then (20) becomes:

$$\alpha \sum_{n=\underline{n}-1}^{N-1} Pr_i(n|G, \vec{\pi}_{-i}, N-1, \alpha)\frac{V-c}{n} = (1-\alpha) \sum_{n=\underline{n}-1}^{N-1} Pr_i(n|B, \vec{\pi}_{-i}, N-1, \alpha)\frac{c}{n}, \tag{B5}$$

and (21) becomes:

$$(1-\alpha) \sum_{n=\underline{n}-1}^{N-1} Pr_i(n|G, \vec{\pi}_{-i}, N-1, \alpha)\frac{V-c}{n} = \alpha \sum_{n=\underline{n}-1}^{N-1} Pr_i(n|B, \vec{\pi}_{-i}, N-1, \alpha)\frac{c}{n}. \tag{B6}$$

Comparing the right-hand sides of (B5) and (B6), we must have

$$\alpha \sum_{n=\underline{n}-1}^{N-1} Pr_i(n|B, \vec{\pi}_{-i}, N-1, \alpha)\frac{c}{n} > (1-\alpha) \sum_{n=\underline{n}-1}^{N-1} Pr_i(n|B, \vec{\pi}_{-i}, N-1, \alpha)\frac{c}{n}, \tag{B7}$$

because $\alpha \in (\frac{1}{2}, 1]$. This then implies the same ordering of the left-hand sides of (B5) and (B6), so

$$(1-\alpha) \sum_{n=\underline{n}-1}^{N-1} Pr_i(n|G, \vec{\pi}_{-i}, N-1, \alpha)\frac{V-c}{n} > \alpha \sum_{n=\underline{n}-1}^{N-1} Pr_i(n|G, \vec{\pi}_{-i}, N-1, \alpha)\frac{V-c}{n}. \tag{B8}$$

However, this provides a contradiction because $\alpha > (1-\alpha)$, meaning only (20) or (21) can hold with equality.

First, consider the case when (21) holds with equality. In this case, (20) is slack, so it is optimal for investors to contribute after observing a good signal. Formally, by Lemma B1 $g_i = 1$ $\forall$ i, so we have that $g_i = g_j$ $\forall$ i, j.

Now suppose, in equilibrium, some investors do not play identical strategies conditional on receiving bad signals. Choose any equilibrium vector $\tilde{\vec{\pi}}_{\hat{B}}$ in which $\exists$ i, j such that $\tilde{b}_i \neq \tilde{b}_j$. Without loss of generality, assume $\tilde{b}_i > \tilde{b}_j$. From (B4), $Pr_i(n|B, \tilde{\vec{\pi}}_{-i}, N-1, \alpha) \neq Pr_j(n|B, \tilde{\vec{\pi}}_{-i}, N-1, \alpha)$

because i incorporates $\tilde{b}_j$ into her probability, while j incorporates $\tilde{b}_i$ into his probability. However, $Pr_i(n|G, \tilde{\vec{\pi}}_{-i}, N-1, \alpha) = Pr_j(n|G, \tilde{\vec{\pi}}_{-i}, N-1, \alpha)$ since $\tilde{g}_i = \tilde{g}_j$ for all investors. As a result, (21) cannot hold with equality for either investor i or investor j (or both). By Lemma B1, either $\tilde{b}_i$ or $\tilde{b}_j$ cannot be an equilibrium strategy, giving a contradiction.

Second, consider the case when (20) holds with equality. In this case, (21) cannot hold, so no investors will invest after receiving a bad signal, so by Lemma B1 $b_i = 0 \; \forall \; i$, so we have that $b_i = b_j$ $\forall \; i, j$.

Now suppose, in equilibrium, some investors do not play identical strategies conditional on receiving good signals. Choose any equilibrium vector $\tilde{\vec{\pi}}_{\hat{G}}$ in which $\exists \; i, j$ such that $\tilde{g}_i \neq \tilde{g}_j$. Without loss of generality, assume $\tilde{g}_i > \tilde{g}_j$. From (B3), $Pr_i(n|G, \tilde{\vec{\pi}}_{-i}, N-1, \alpha) \neq Pr_j(n|G, \tilde{\vec{\pi}}_{-i}, N-1, \alpha)$ because i incorporates $\tilde{g}_j$ into her probability, while j incorporates $\tilde{g}_i$ into his probability. However, $Pr_i(n|B, \tilde{\vec{\pi}}_{-i}, N-1, \alpha) = Pr_j(n|B, \tilde{\vec{\pi}}_{-i}, N-1, \alpha)$ since $\tilde{b}_i = \tilde{b}_j$ for all investors. As a result, (21) cannot hold with equality for either investor i or investor j (or both). By Lemma B1, either $\tilde{g}_i$ or $\tilde{g}_j$ cannot be an equilibrium strategy, giving a contradiction.

$\blacksquare$

The intuition for the lemma is simple. For mixing strategies to form an equilibrium, it must be the case that either (20) or (21) binds for all investors. However, if any investor plays a different mixing strategy, the probability of realizing state n must be different for that investor relative to all other investors because conditional probabilities are formed taking all other investors' strategies into consideration. As a result, the non-identical mixing strategy cannot be an equilibrium as either (20) or (21) would not bind for some investor.

For a given $\{N, n\}$, we use Lemma B1 and Lemma B2 and numerically solving for the values g^* and b^* which satisfy (20) and (21). After solving for g^* and b^*, we then calculate projects' value-add based on the probabilities of realizing each state $n \in [\underline{n}, N]$ for both good and bad projects and the associated costs and payoffs in each state.